



# $S_3$ Higgs Potential and Texture zeros in Supersymmetric standard model

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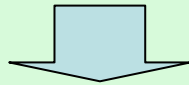
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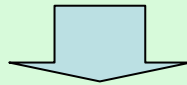
# 1, Introduction

Texture-zeros in quark–lepton mass matrix are successful to predict masses and mixings.

$$M = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$



**However**, the origin of zero is **not clear!**



**Discrete flavor symmetry approach**

The favorable mass matrix structure can be derived when certain **Yukawa couplings** are forbidden by discrete symmetry.



When we consider **small discrete flavor Symmetry**  $S_3$ , Texture-zeros can be derived??

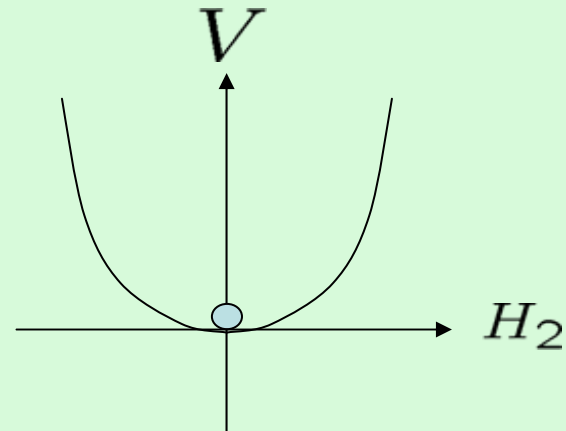
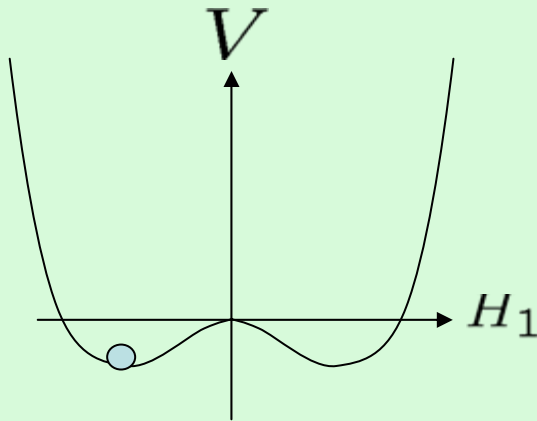
$S_3$  : permutations of three objects

Mass matrix

$$M_{ij} = y_{ij} \langle H \rangle$$



To derive Texture-zeros, we consider **VEV=0**.





Higgs : All representations are taken into account.

$S_3$  irreducible representation  $\underline{2}$   $\underline{1}_S$   $\underline{1}_A$



To analyze whether **VEV = 0** is possible or not,

Higgs potential analysis  
on SUSY



# **contents**

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3, 4,  $S_3$  invariant Higgs scalar potential  
analysis (**VEV=0**)

5, Phenomenology

6, Discussion



# 2, $S_3$ invariant mass matrix on SUSY

N. Haba and K. Yoshioka, hep-ph/051118

Tensor product of  $S_3$  doublet

$\phi, \psi : S_3$  doublet (complex representation)

$$\phi \times \psi = \underbrace{(\phi_1^* \psi_2, \phi_2^* \psi_1)^T}_{\underline{2}} + \underbrace{(\phi_1^* \psi_1 - \phi_2^* \psi_2)}_{\underline{1}_A} + \underbrace{(\phi_1^* \psi_1 + \phi_2^* \psi_2)}_{\underline{1}_S}$$

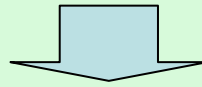
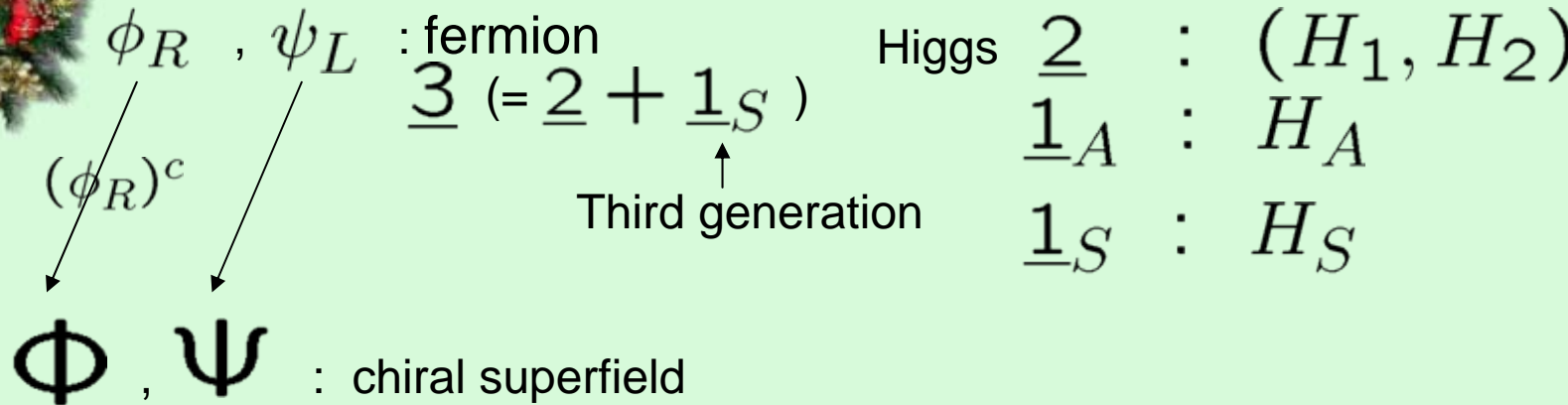
$$\phi^c \times \psi = \underbrace{(\phi_2 \psi_2, \phi_1 \psi_1)^T}_{\underline{2}} + \underbrace{(\phi_1 \psi_2 - \phi_2 \psi_1)}_{\underline{1}_A} + \underbrace{(\phi_1 \psi_2 + \phi_2 \psi_1)}_{\underline{1}_S}$$

$\phi^c \equiv \sigma_1 \phi^* : S_3$  doublet

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$



# Supersymmetric $S_3$ mass matrix



## Supersymmetric Dirac mass matrix

$$W = \Phi M_D \Psi$$

$$M_D = \left( \begin{array}{cc|c} aH_1 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & dH_1 \\ \hline eH_2 & eH_1 & fH_S \end{array} \right)$$

## Supersymmetric Majorana mass matrix

$$W = \Phi M_R \Phi$$

$$M_R = \left( \begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array} \right)$$





# 3, $S_3$ invariant Higgs scalar potential

## superfields

$\hat{H}_{uS}, \hat{H}_{dS}$  :  $S_3$  symmetric singlet

$\hat{H}_{uA}, \hat{H}_{dA}$  :  $S_3$  antisymmetric singlet

$\hat{H}_{u1}, \hat{H}_{d1}, \hat{H}_{u2}, \hat{H}_{d2}$  :  $S_3$  doublet

hat : superfield

## gauge groups

$U(1)_Y$  ; up type Higgs :  $\frac{1}{2}$ , down type Higgs :  $-\frac{1}{2}$

$SU(2)_L$  ; all Higgses are  $SU(2)_L$  doublet



# Higgs potential

$$\begin{aligned}
 V &= V_{\text{SUSY}} + V_{\text{soft}} \\
 &= (|\mu_S|^2 + m_{uS}^2) v_{uS}^2 + (|\mu_S|^2 + m_{dS}^2) v_{dS}^2 - 2b_S v_{uS} v_{dS} \\
 &\quad + (|\mu_A|^2 + m_{uA}^2) v_{uA}^2 + (|\mu_A|^2 + m_{dA}^2) v_{dA}^2 - 2b_A v_{uA} v_{dA} \\
 &\quad + (|\mu_D|^2 + m_{uD}^2) v_{u1}^2 + (|\mu_D|^2 + m_{dD}^2) v_{d2}^2 - 2b_D v_{u1} v_{d2} \\
 &\quad + (|\mu_D|^2 + m_{uD}^2) v_{u2}^2 + (|\mu_D|^2 + m_{dD}^2) v_{d1}^2 - 2b_D v_{u2} v_{d1} \\
 &\quad + \frac{2}{g_Y^2 + g_2^2} A^2
 \end{aligned}$$

$$A \equiv \frac{g_Y^2 + g_2^2}{4} \{ (v_{uS}^2 - v_{dS}^2) + (v_{uA}^2 - v_{dA}^2) + (v_{u1}^2 - v_{d2}^2) + (v_{u2}^2 - v_{d1}^2) \}$$

$$v_{(u,d)S} \equiv |\langle H_{(u,d)S}^0 \rangle|, \quad v_{(u,d)A} \equiv |\langle H_{(u,d)A}^0 \rangle|, \quad v_{(u,d)1} \equiv |\langle H_{(u,d)1}^0 \rangle|, \quad v_{(u,d)2} \equiv |\langle H_{(u,d)2}^0 \rangle|$$

\* assumption      Charged Higgs VEVs are zero.



# 4, Potential analysis

## Equations at vacua

$$S \left( (|\mu_S|^2 + m_{uS}^2) v_{uS} = b_S v_{dS} - A v_{uS} \quad (|\mu_S|^2 + m_{dS}^2) v_{dS} = b_S v_{uS} + A v_{dS} \right)$$

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$$A \left( (|\mu_A|^2 + m_{uA}^2) v_{uA} = b_A v_{dA} - A v_{uA} \quad (|\mu_A|^2 + m_{dA}^2) v_{dA} = b_A v_{uA} + A v_{dA} \right)$$

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$$D \left( \begin{aligned} (|\mu_D|^2 + m_{uD}^2) v_{u1} &= b_D v_{d2} - A v_{u1} & (|\mu_D|^2 + m_{dD}^2) v_{d2} &= b_D v_{u1} + A v_{d2} \\ (|\mu_D|^2 + m_{uD}^2) v_{u2} &= b_D v_{d1} - A v_{u2} & (|\mu_D|^2 + m_{dD}^2) v_{d1} &= b_D v_{u2} + A v_{d1} \end{aligned} \right)$$

$$A \equiv \frac{1}{4}(g_Y^2 + g_2^2) \left\{ (v_{uS}^2 - v_{dS}^2) + (v_{uA}^2 - v_{dA}^2) + (v_{u1}^2 - v_{d2}^2) + (v_{u2}^2 - v_{d1}^2) \right\}$$



# 4.1, Classification of vacua about $S_3$ **symmetric singlet**, $S_3$ **antisymmetric singlet**

\* Both up-type and down-type Higgs VEVs are zero

This solution always exists.

$$v_{uS} = v_{dS} = 0$$

$$v_{uA} = v_{dA} = 0$$

$$(|\mu_S|^2 + m_{uS}^2) v_{uS} = b_S v_{dS} - A v_{uS} \quad (|\mu_S|^2 + m_{dS}^2) v_{dS} = b_S v_{uS} + A v_{dS}$$

\* One of up-type and down-type Higgs VEVs is zero

$$v_{uS} = 0 \quad v_{dS} \neq 0 \quad \text{or} \quad v_{uS} \neq 0 \quad v_{dS} = 0$$

$b_S = 0$  is **necessary** for this solution to exist.

For antisymmetric singlet  $S \rightarrow A$



# Proof

When  $b_S = 0$ , the following solutions exist.

		condition
$v_{uS} = 0$	$v_{dS} = 0$	
$( \mu_S ^2 + m_{uS}^2) + A _{v_{dS}=0} = 0$	$v_{dS} = 0$	
$v_{uS} = 0$	$( \mu_S ^2 + m_{dS}^2) - A _{v_{uS}=0} = 0$	
$( \mu_S ^2 + m_{uS}^2) + A = 0$		$2 \mu_S ^2 + m_{uS}^2 + m_{dS}^2 = 0$

$$A \equiv \frac{g_Y^2 + g_2^2}{4} \left\{ (v_{uS}^2 - v_{dS}^2) + (v_{uA}^2 - v_{dA}^2) + (v_{u1}^2 - v_{d2}^2) + (v_{u2}^2 - v_{d1}^2) \right\}$$

The solution that **one of two VEVs is zero** is contained.

Oppositely, when  $v_{uS} = 0$ ,  $v_{dS} \neq 0$ ,

$$b_S = 0 \quad (|\mu_S|^2 + m_{dS}^2) - A|_{v_{uS}=0} = 0$$

Therefore  $b_S = 0$  is **the necessary condition** of  $v_{uS} = 0$   $v_{dS} \neq 0$  or  $v_{uS} \neq 0$   $v_{dS} = 0$ .

# 4,2 Classification of vacua

## about $S_3$ doublet

Because  $b_D$  is common to  $v_{u1}, v_{d2}$  and  $v_{u2}, v_{d1}$ , it is necessary to discuss **four VEVs simultaneously**.

Independent candidates of **VEV=0** are below.

# **all of VEVs are zero**

$$v_{u1} = 0, v_{d2} = 0, v_{u2} = 0, v_{d1} = 0$$

$b_D$  dependence

# **three of VEVs are zero**

$$v_{u1} = 0, v_{d2} = 0, v_{u2} = 0, v_{d1} \neq 0$$

$$b_D = 0$$

# **two of VEVs are zero**

$$v_{u1} = 0, v_{d2} = 0, v_{u2} \neq 0, v_{d1} \neq 0$$

$$v_{u1} = 0, v_{d2} \neq 0, v_{u2} = 0, v_{d1} \neq 0$$

$$b_D = 0$$

$\triangle$   $v_{u1} = 0, v_{d2} \neq 0, v_{u2} \neq 0, v_{d1} = 0$

$$b_D = 0$$

# **one of VEVs is zero**

$\triangle$   $v_{u1} \neq 0, v_{d2} \neq 0, v_{u2} \neq 0, v_{d1} = 0$

$$b_D = 0$$

# **all of VEVs are non zero**

$$v_{u1} \neq 0, v_{d2} \neq 0, v_{u2} \neq 0, v_{d1} \neq 0$$





\* Why are the followings not taken?

$$v_{u1} = 0, v_{d2} \neq 0, v_{u2} \neq 0, v_{d1} = 0$$

$$v_{u1} \neq 0, v_{d2} \neq 0, v_{u2} \neq 0, v_{d1} = 0$$

These solutions satisfy  $2|\mu_D|^2 + m_{uD}^2 + m_{dD}^2 = 0$  .  
 $b_D = 0$



The condition could be satisfied **at certain energy scales** but RGE (Renormalization Group Equation) dependent.



**It is difficult to construct models that satisfy that condition.**

# Summary of potential analysis



symmetric singlet

$$* v_{uS} = v_{dS} = 0$$

$$* v_{uS} \neq 0, v_{dS} \neq 0$$

antisymmetric singlet

$$* v_{uA} = v_{dA} = 0$$

$$* v_{uA} \neq 0, v_{dA} \neq 0$$

doublet

$v_{u1}$	$v_{d2}$	$v_{u2}$	$v_{d1}$
0	0	0	0
0	0		
		0	0

Consequently, there are **14 patterns** in terms of Texture-zeros.

$$: (2 \times 2 \times 4) - 2$$

assumption

$$b_S \neq 0, b_A \neq 0, b_D \neq 0$$



# 5, Phenomenology

## Quark / Lepton mass matrix structure

\* left handed neutrino mass matrix analysis

Supersymmetric Dirac mass matrix

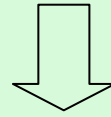
$$M_{(u,d)D} = \left( \begin{array}{cc|c} av_{(u,d)1} & bv_{(u,d)S} + cv_{(u,d)A} & dv_{(u,d)2} \\ bv_{(u,d)S} - cv_{(u,d)A} & av_{(u,d)2} & dv_{(u,d)1} \\ \hline ev_{(u,d)2} & ev_{(u,d)1} & fv_{(u,d)S} \end{array} \right)$$

Supersymmetric Majorana mass matrix

$$M_R = \left( \begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array} \right)$$

## see-saw mechanism

$$-M_\nu = M_D^T M_R^{-1} M_D$$



**As results of potential analysis, there are two interesting solutions in terms of Texture-zeros.**

$v_{uS}$	$v_{dS}$	$v_{uA}$	$v_{dA}$	$v_{u1}$	$v_{d2}$	$v_{u2}$	$v_{d1}$
0	0			0	0		

$$M_u = \begin{pmatrix} 0 & cv_{uA} & dv_{u2} \\ -cv_{uA} & av_{u2} & 0 \\ ev_{u2} & 0 & 0 \end{pmatrix} M_{d,e} = \begin{pmatrix} a'v_{d1} & c'v_{dA} & 0 \\ -c'v_{dA} & 0 & d'v_{d1} \\ 0 & e'v_{d1} & 0 \end{pmatrix} M_\nu = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

$v_{uS}$	$v_{dS}$	$v_{uA}$	$v_{dA}$	$v_{u1}$	$v_{d2}$	$v_{u2}$	$v_{d1}$
0	0					0	0

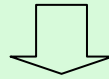
$$M_u = \begin{pmatrix} av_{u1} & cv_{uA} & 0 \\ -cv_{uA} & 0 & dv_{u1} \\ 0 & ev_{u1} & 0 \end{pmatrix} M_{d,e} = \begin{pmatrix} 0 & c'v_{dA} & d'v_{d2} \\ -c'v_{dA} & a'v_{d2} & 0 \\ e'v_{d2} & 0 & 0 \end{pmatrix} M_\nu = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$$



# 6, Discussion

## \* **Problem**

Global symmetry  $\rightarrow$  massless Higgs??



- Higgs mass spectrum
- $S_3$  breaking in Lagrangian

## \* **Future work**

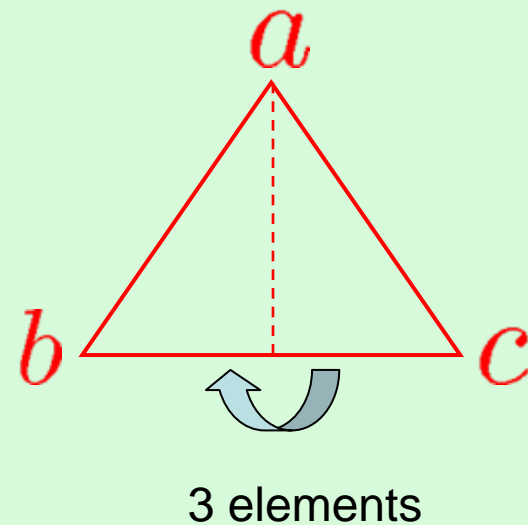
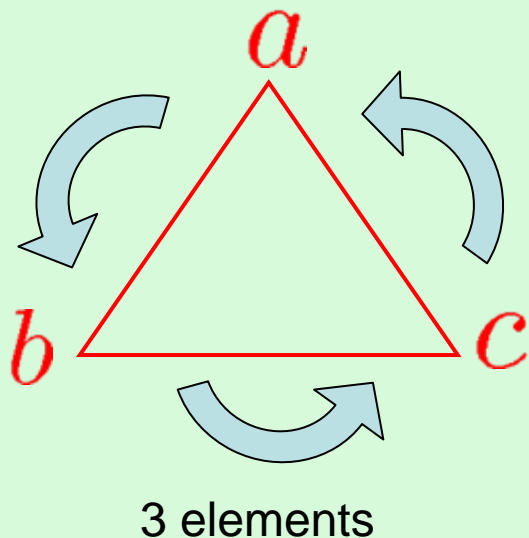
- $S_{3L} \times S_{3R}$  symmetry







$S_3$  : *permutations of three objects*  
( $a, b, c$ )



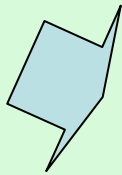
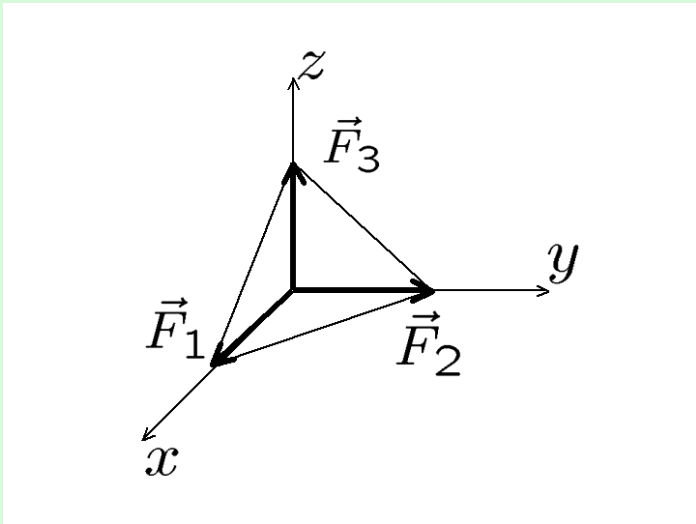
$S_3$  is the smallest group of non abelian finite groups.





# $S_3$ irreducible representation

$$\underline{2} \quad \underline{1}_S \quad \underline{1}_A$$



$$\underline{3} = \underline{2} + \underline{1}_S$$

