Non-Abelian Discrete Family Symmetry to Soften SUSY Flavor Problem and to Suppress Proton Decay

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1. Introduction

Flavor symmetries can determine the structure of the Yukawa couplings(S3,S4,D4,...).

We consider Q6 symmetric model.

In this talk, we will show the flavor symmetry can work to suppress

1. SUSY FCNCs

2. dim.5 proton decay.

Yukawa coupling (with R-Parity)

$$W = Y_u QUH_u + Y_d QDH_d + Y_e LEH_d + \mu H_u H_d + Y_N LNH_u + \frac{1}{2} M_R NN$$

VEV $\langle H_u \rangle = v_u, \langle H_d \rangle = v_d, v_u^2 + v_d^2 \equiv v^2 = (174\text{GeV})$

$$U_{uL}^\dagger m_u U_{uR} = m_u^{\text{diag}} = \text{diag}(m_u, m_c, m_t)$$

$$\vdots$$

$$U_\nu^T m_\nu U_\nu = m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$$

where $m_\nu = m_{N_D} M_R^{-1} m_{N_D}^T$

$$V_{CKM} = U_{uL}^\dagger U_{dL}, V_{MNS} = U_{eL}^\dagger U_\nu$$

Soft SUSY Breaking Terms

$$\begin{aligned} V_{soft} &= \frac{1}{2} M_i \lambda_i \lambda_i + c.c && \text{gaugino masses} \\ &+ h_u Q U H_u + \cdots c.c. && \text{A-terms} \\ &+ Q^\dagger m^2_{\tilde{Q}} Q + \cdots && \text{scalar masses} \\ &+ b H_u H_d + c.c. && \text{B-term} \end{aligned}$$

The A-terms and the scalar masses can not be diagonalized by $U_{uL}, U_{dL}, \cdots$ in general.

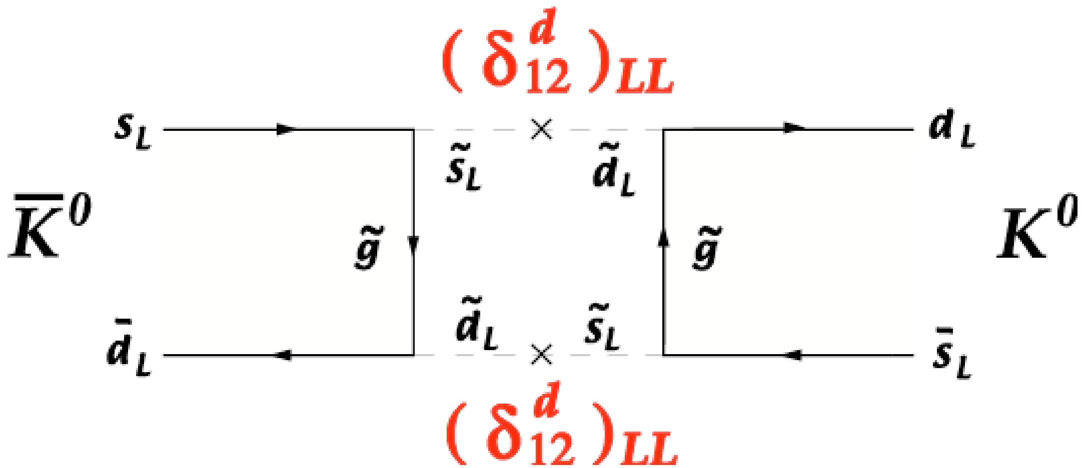
→ Flavor Changing Neutral Currents (FCNCs)

SUSY Flavor Problem

Mass Insertion Parameter

$$\left(\delta^d_{ij}\right)_{LL} = \frac{\left(U^\dagger_{dL} m^2_{\tilde{Q}} U_{dL}\right)_{ij}}{m^2_{\tilde{q}}}$$

K^0- $\bar{K}^0$ mixingを考える



$$\Delta m_K \propto \frac{m_K f_K^2}{m^2_{\tilde{q}}} \left(\delta^d_{12}\right)^2_{LL} < 3.5 \times 10^{-12} \mathrm{MeV} = \Delta m_K^{exp}$$

これより δ のExperimental boundが得られる。

$$\left|\left(\delta^d_{12}\right)_{LL}\right| < 4.0 \times 10^{-2}$$ Masiero et.al.(1996)

Conditions to suppress FCNCs

Off-diag. elements of δ generate FCNCs.

1) Alignment

$$h_d \propto Y_d, \quad m^2_{\tilde{U}} \propto Y_u Y_u^\dagger, \quad \text{etc.}$$

$$\rightarrow U^\dagger_{dL} h_d U_{dR} = \text{diag}, \quad U^\dagger_{uL} M^2_{\tilde{U}} U_{uL} = \text{diag}$$

2) Degeneracy(Universality)

$$m^2_{\tilde{Q}} \propto 1 \quad \rightarrow \quad U^\dagger_{uL} m^2_{\tilde{Q}} U_{uL} \propto 1$$

3) Decoupling

$$m_{\tilde{Q}}, \ m_{\tilde{d}} > \mathcal{O}(10\text{TeV})$$

1) and 2) can be realized by flavor symmetry

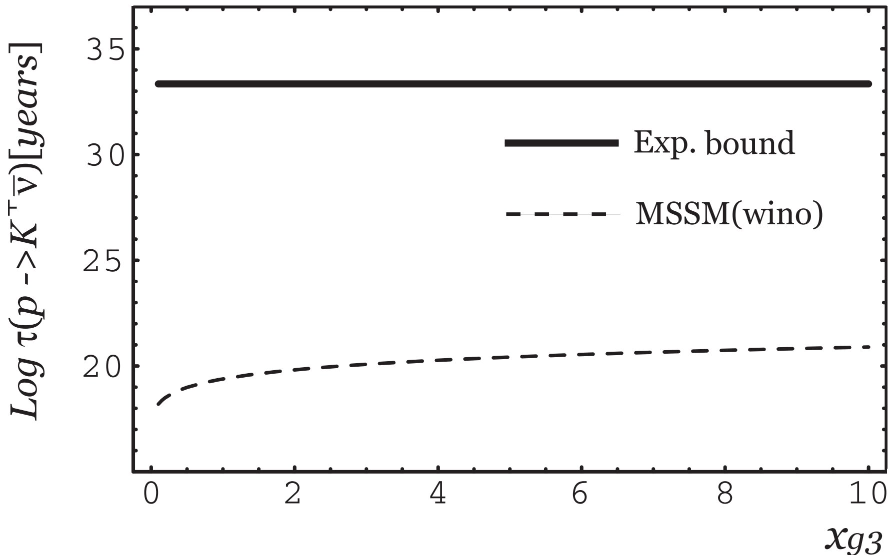
Proton Decay

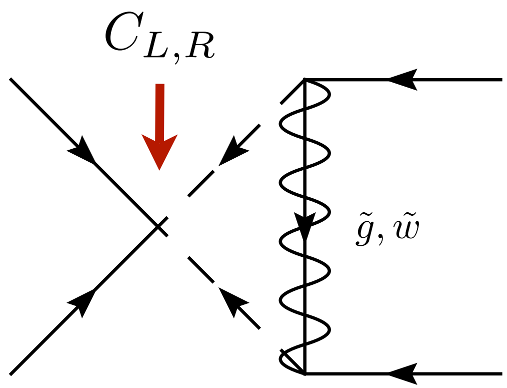
Baryon and lepton number violating dim. 5 operator can be generated at the Planck scale.

$$W_5 = \frac{1}{M_{PL}} \left[C_L^{ijkl} Q_i Q_j Q_k L_l + C_R^{ijkl} U_i D_j U_k E_l \right]$$

From the experimental bound of proton lifetime

$$\tau(p \to K^+ \bar{\nu})^{\exp} = 2 \times 10^{33} \text{years} \longrightarrow C_{L,R} < 10^{-7}$$





Example : U(1)$_H$ horizontal symmetry

- SUSY FCNC can be suppressed by Alignment.
- Smallness of the coefficients $C_{L,R}$ is related to the smallness of Yukawa couplings.

$$H(\Phi) \quad : \quad U(1)_H \text{ charge of field } \Phi$$

$$\begin{aligned} W_Y &= \left(\frac{S}{M}\right)^{H(Q_i)+H(U_j)+H(H_u)} Q_i U_j H_u + \left(\frac{S}{M}\right)^{H(Q_i)+H(D_j)+H(H_d)} Q_i D_j H_d \\ &+ \left(\frac{S}{M}\right)^{H(L_i)+H(E_j)+H(H_d)} L_i E_j H_d \end{aligned}$$

U(1)$_H$ is broken at the scale M by VEV of the flavon S .

$$\frac{\langle S \rangle}{M} \sim \lambda \sim 0.2 \quad \Rightarrow \quad Y_u^{ij} \sim \lambda^{H(Q_i)+H(U_j)+H(H_u)}$$

A-terms

$$v_u A_u \sim m_{SUSY} m_u, \text{ etc.}$$: Alignment

Scalar mass

$$\left(m^2_{\tilde{Q}}\right)_{ij} \sim m^2_{SUSY} \lambda^{|-H(Q_i)+H(Q_j)|}, \text{ etc.}$$

: Small off-diag. elements

For non-Abelian groups,

Scalar mass

$$m^2_{\tilde{\phi}} \phi^\dagger_I \phi_I, \ I = 1, 2(, 3)$$: Degeneracy

Dim. 5 operators

$$W_5 = \frac{1}{M_{PL}}\left[\left(\frac{S}{M}\right)^{H(Q_i)+H(Q_j)+H(Q_k)+H(L_l)} Q_iQ_jQ_kL_l + \left(\frac{S}{M}\right)^{H(U_i)+H(D_j)+H(U_k)+H(E_l)}\right] U_iD_jU_kE_l$$

small Yukawa → small $C_{L,R}$

Examples of Flavor symmetries to overcome SUSY Flavor Problem and Proton Decay

$\Delta(75)$	Murayama and Kaplan(1994)
$(S_3)^3$	Carone, Hall and Murayama(1996)
$U(1)$	Ben-Hamo and Nir(1994)
	Kakizaki and Yamaguchi(2002)
	Harnik, Larson, Murayama and Thormeier(2005)

We consider Q6 symmetric model

2. The Supersymmetric Model with Q6 Flavor Symmetry

Babu and Kubo(2005)

- Assuming three “generations” of Higgs doublets.
- Every three generations belong to some Q6 irreps.

$$\mathbf{2} \quad : \quad Q_I \quad (I = 1, 2)$$

$$\mathbf{2}' \quad : \quad U_I^c, D_I^c, L_I, E_I^c, N_I^c, H_I^u, H_I^d$$

$$\mathbf{1}' \quad : \quad Q_3$$

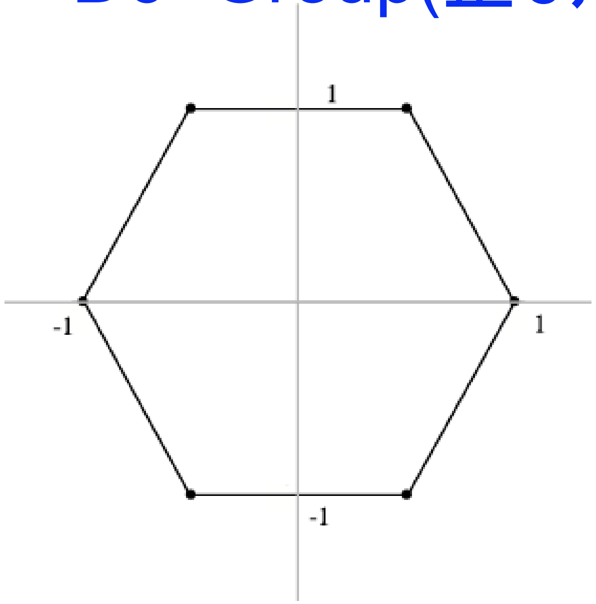
$$\mathbf{1}''' \quad : \quad U_3^c, D_3^c, H_3^u, H_3^d$$

$$\mathbf{1} \quad : \quad L_3, E_3^c$$

$$\mathbf{1}'' \quad : \quad N_3^c$$

Q6 Group

D6 Group(正6角形の対称群)



$$A = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \quad \phi = \frac{2\pi}{6}$$

$$B = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

D6群のelementは行列A,Bを使って表される。

$$D_6 = \left\{1, A, A^2, ..., A^5, B, AB, ..., A^5B\right\}$$

既約表現は

$$\mathbf{2}, \mathbf{2}', \mathbf{1}_{(+,+)}, \mathbf{1}'_{(+,-)}, \mathbf{1}''_{(-,+)}, \mathbf{1}'''_{(-,-)}$$

Q6 Group

D6とはBの行列が異なっている。

$$A = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \quad \phi = \frac{2\pi}{6}$$

$$B = \begin{pmatrix} i & \\ & -i \end{pmatrix}$$

element,既約表現は

$$D_6 = \{1, A, A^2, ..., A^5, B, AB, ..., A^5B\}$$

$$\mathbf{2}, \mathbf{2}', \mathbf{1}_{(+,+)}, \mathbf{1}'_{(+,-)}, \mathbf{1}''_{(-,-\mathbf{i})}, \mathbf{1}'''_{(-,+\mathbf{i})}$$

Q6 symmetric Superpotential

$$
\begin{aligned}
W_D &= Y_a^d Q_3 D_3^c H_3^d + Y_b^d Q_I \left(\sigma^1\right)_{IJ} D_3^c H_J^d \\
&- Y_{b'}^d Q_3 D_I^c \left(i\sigma^2\right)_{IJ} H_J^d + Y_c^d Q_I \left(\sigma^1\right)_{IJ} D_J^c H_3^d, \\
W_E &= Y_b^e L_I E_3^c H_I^d + Y_{b'}^e L_3 E_I^c H_I^d \\
&+ Y_c^e f_{IJK} L_I E_J^c H_K^d, \quad 1 = -f_{111} = f_{122} = f_{212} = f_{221} \\
\cdots
\end{aligned}
$$

Where all Ys are assumed to be real.

⇩

Spontaneous CP violation

$$\langle H_1^d \rangle = \langle H_2^d \rangle = v_1^d e^{i\theta_1^d} \ , \ \langle H_3^d \rangle = v_3^d e^{i\theta_3^d}$$

Realistic masses and mixings can be obtained from the following parameters.

$$
\begin{aligned}
M_{u,d} &= m_{t,b} \begin{pmatrix} 0 & q_{u,d}/y_{u,d} & 0 \\ -q_{u,d}/y_{u,d} & 0 & b_{u,d} \\ 0 & b'_{u,d} & y^2_{u,d} \end{pmatrix}, \\
q_u &= 0.0002260, b_u = 0.04596, b'_u = 0.08959, y_u = 0.99746 \\
q_d &= 0.005110, b_d = 0.02609, b'_d = 0.7682, y_d = 0.8000, \\
M_e &= m_\tau \begin{pmatrix} -y^e_2 & y^e_2 & y^e_5 \\ y^e_2 & y^e_2 & y^e_5 \\ y^e_4 & y^e_4 & 0 \end{pmatrix}, \qquad \begin{matrix} y^e_2 = 0.04206 \\ y^e_4 = 0.0002893 \\ y^e_5 = 0.7057 \end{matrix}
\end{aligned}
$$

The mixing matrices are obtained as follows:

$$
\begin{aligned}
|U_{uL}| &= \begin{pmatrix} 0.7446 & 0.6667 & 0.03238 \\ 0.6675 & 0.7439 & 0.03235 \\ 0.002518 & 0.04571 & 0.9990 \end{pmatrix}, \\
|U_{dL}| &= \begin{pmatrix} 0.6167 & 0.7871 & 0.009848 \\ 0.7872 & 0.6166 & 0.01436 \\ 0.007876 & 0.01553 & 0.9998 \end{pmatrix}, \\
|U_{eL}| &= \begin{pmatrix} 0.003427 & 0.7071 & 0.7071 \\ 0.003452 & 0.7071 & 0.7071 \\ 1.000 & 0.004864 & 0.00001722 \end{pmatrix}.
\end{aligned}
$$

3.SUSY FCNC constraints

Q6 invariant Soft SUSY Breaking Terms

scalar mass

$$m_1^2 \phi_I^\dagger \phi_I + m_3^2 \phi_3^\dagger \phi_3$$

$$m_1^2 - m_3^2 \equiv m_{\tilde{a}}^2 \Delta_a \neq 0$$

$$(a = Q, u, d, e)$$

A-term

$$y_i \to y_i A_i$$

$$A_i - A_j \neq 0$$

Real A terms

↓

NO SUSY CP Problem

Degeneracy and alignment are partially realized

by the flavor symmetry.

Kaplan and Schmaltz(1994), Nir and Seiberg(1993)
Dine, Leigh and Kagan(1993)

Mass insertion parameter δ depends on Δ_a and $A_i - A_j$

$$\left(\delta_{ij}^d\right)_{LL} = \frac{\left(U_{dL}^\dagger m_{\tilde{Q}}^2 U_{dL}\right)_{ij}}{m_{\tilde{q}}^2} \text{ etc.}$$

	Exp. bound	Q_6 Model
$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{LL,RR}\rvert}$	$4.0\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$(LL)1.2\times 10^{-4}\Delta_Q, (RR)1.7\times 10^{-1}\Delta_d$
$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$	$2.8\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$4.5\times 10^{-3}\sqrt{\Delta_Q\Delta_d}$
$\sqrt{\lvert\mathrm{Re}(\delta^d_{12})^2_{LR}\rvert}$	$4.4\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4}\tilde{m}_{\tilde{q}}^{-1}$
★ $\sqrt{\lvert\mathrm{Re}(\delta^d_{13})^2_{LL,RR}\rvert}$	$9.8\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$(LL)7.9\times 10^{-3}\Delta_Q, (RR)1.4\times 10^{-1}\Delta_d$ ★
$\sqrt{\lvert\mathrm{Re}(\delta^d_{13})_{LL}(\delta^d_{13})_{RR}\rvert}$	$1.8\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$3.4\times 10^{-2}\sqrt{\Delta_Q\Delta_d}$
$\sqrt{\lvert\mathrm{Re}(\delta^d_{13})^2_{LR}\rvert}$	$3.3\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4}\tilde{m}_{\tilde{q}}^{-1}$
$\sqrt{\lvert\mathrm{Re}(\delta^u_{12})^2_{LL,RR}\rvert}$	$1.0\times 10^{-1}\ \tilde{m}_{\tilde{q}}$	$(LL)1.2\times 10^{-4}\Delta_Q, (RR)4.4\times 10^{-4}\Delta_u$
$\sqrt{\lvert\mathrm{Re}(\delta^u_{12})_{LL}(\delta^u_{12})_{RR}\rvert}$	$1.7\times 10^{-2}\ \tilde{m}_{\tilde{q}}$	$2.3\times 10^{-4}\sqrt{\Delta_Q\Delta_u}$
$\sqrt{\lvert\mathrm{Re}(\delta^u_{12})^2_{LR}\rvert}$	$3.1\times 10^{-3}\ \tilde{m}_{\tilde{q}}$	$\sim 10^{-4}\tilde{m}_{\tilde{q}}^{-1}$
$\lvert(\delta^d_{23})_{LL,RR}\rvert$	$8.2\ \tilde{m}_{\tilde{q}}^2$	$(LL)1.6\times 10^{-2}\Delta_Q, (RR)4.7\times 10^{-1}\Delta_d$
$\lvert(\delta^d_{23})_{LR}\rvert$	$1.6\times 10^{-2}\ \tilde{m}_{\tilde{q}}^2$	$\sim 10^{-2}\tilde{m}_{\tilde{q}}^{-1}$

Table 1: Experimental bounds on δ's, where the parameter $\tilde{m}_{\tilde{q}}$ denote $m_{\tilde{q}}/500$ GeV.

The degeneracy conditions

$$\Delta_Q < 10,\ \ \Delta_d < 10^{-1},\ \ \Delta_u < 10^{+3}$$

4. Suppression of proton decay

dim.5 operators $(\Delta B \neq 0, \Delta L \neq 0)$ at M_{PL}
are controlled by a flavor symmetry.

Murayama and Kaplan(1994), Carone, Hall and Murayama(1996)
Ben-Hamo and Nir(1994),Kakizaki and Yamaguchi(2002)
Harnik, Larson, Murayama and Thormeier(2005)

⇒ Q6 symmetric dim.5 operators

$$W_5 = \frac{1}{M_{PL}} [C_L Q_I Q_I Q_3 L_3$$

$$+ C_R^{(1)} E_I^c \left(\sigma^1\right)_{IJ} U_J^c U_3^c D_3^c + C_R^{(2)} E_I^c \left(\sigma^1\right)_{IJ} U_1^c U_2^c D_J^c]$$

$$\Phi_3^f = V_{32} \Phi_2^m$$

small suppression factor

=0(gluino dressing)

cf.

$$W_5^{MSSM} = \frac{1}{M_{PL}} C_L Q_1 Q_1 Q_2 L_i$$

no suppression factor

The properties of Q6 symmetric dim.5 operator

i) Suppression by small mixing parameters

$$\Phi_3^f = V_{32}\Phi_2^m \quad \longrightarrow \quad (U_{dL})_{32} = 0.01553$$

ii) Enhancement by gluino dressing diagrams

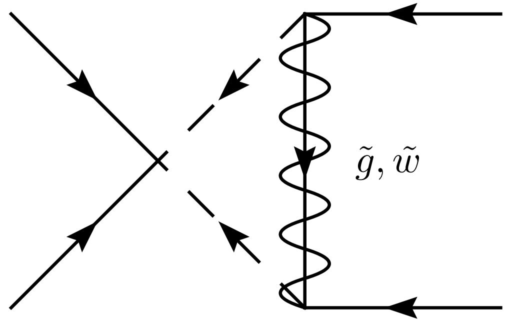
$$\Delta_Q < 10,\ \Delta_d < 10^{-1},\ \Delta_u < 10^{+3}$$

cf. In usual (MSSM,GUTs,etc.)models,

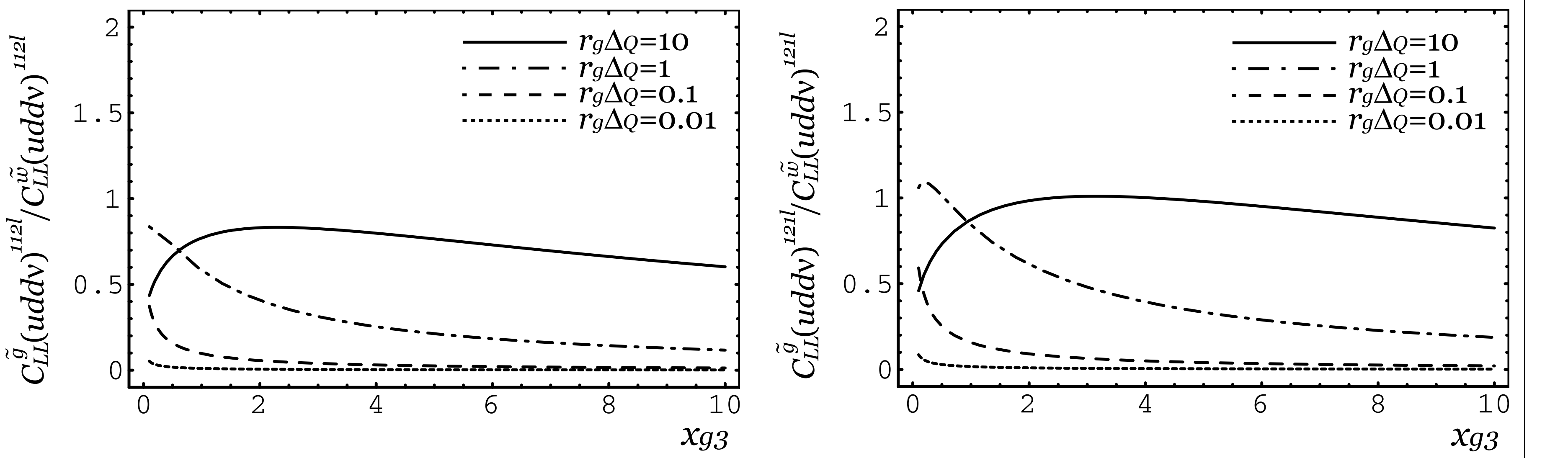
small FCNC

→degenerated squark mass

→negligible gluino contribution



ii)The ratio of the gluino and wino contribution to $p \to K^+ \bar{\nu}$



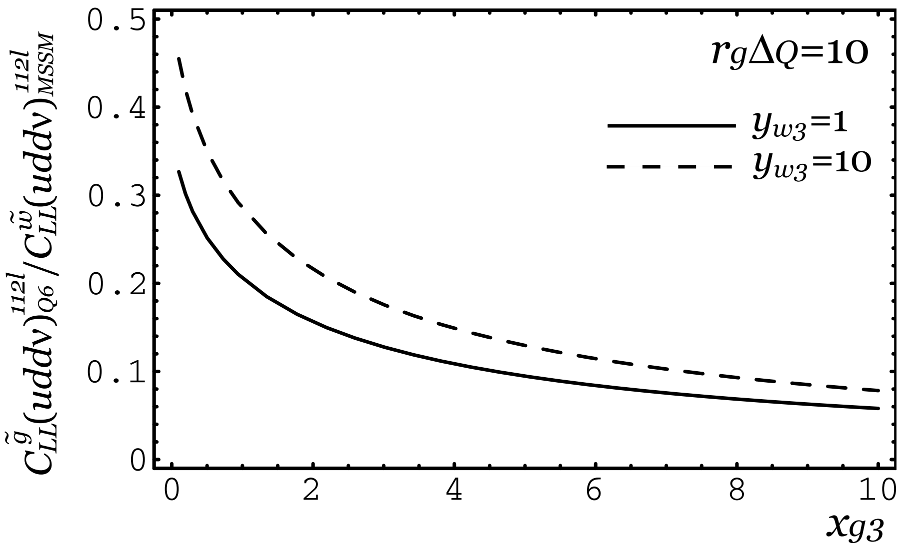
Where
$$x_{g3} = \frac{m^2_{\tilde{Q}3}}{m^2_{\tilde{g}}},\ y_{w3} = \frac{m^2_{\tilde{L}3}}{m^2_{\tilde{w}}},\ r_g \Delta_Q = \frac{m^2_{\tilde{Q}}}{m^2_{\tilde{g}}} \frac{m^2_{\tilde{Q}1} - m^2_{\tilde{Q}3}}{m^2_{\tilde{Q}}},\ m_{\tilde{w}} = 0.27 m_{\tilde{g}},$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{(4\pi)^2 M_{PL}} \left[\sum_{\substack{M=d,s \\ \ell=e,\mu}} C_{LL}(udue)^{1M1\ell}(ud^M)(ue^\ell) + \sum_{\substack{M,N=d,s \\ \ell=e,\mu,\tau}} C_{LL}(udd\nu)^{1MN\ell}(ud^M)(d^N\nu^\ell) \right]$$

→ Gluino dressing can be comparable to wino.

→ How do the mixings work?

i) The ratio of gluino in Q6 model and wino in MSSM



$$C^{\tilde{g}}_{LL}(udd\nu)^{1MN\ell}_{Q6} = 4\pi\frac{4\alpha_3}{3}\sum_{I=1,2} U^{I1}_{uL}U^{IM}_{dL}U^{3N}_{dL}U^{3\ell}_{eL}\left(F^{\tilde{g}}(x_{g1},x_{g3}) - F^{\tilde{g}}(x_{g1},x_{g1})\right)$$

$$C^{\tilde{w}}_{LL}(udd\nu)^{112\ell}_{MSSM} = 4\pi\alpha_2\cos^2\theta_c\left(F^{\tilde{w}}(x_{w1},y_{w3}) + F^{\tilde{w}}(x_{w1},x_{w1})\right)$$

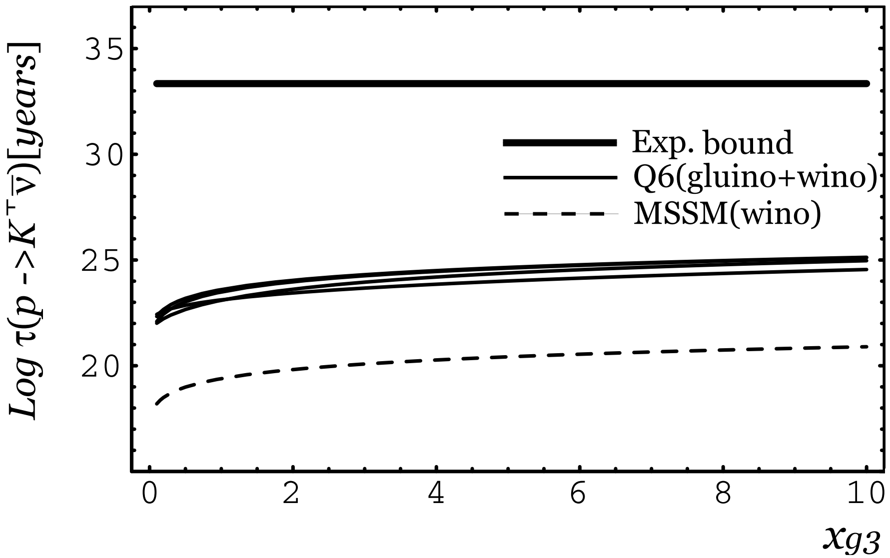
and the loop function is

$$F^{\tilde{g}}(x_{g1},x_{g3}) = \frac{1}{m_{\tilde{g}}}\frac{1}{x_{g1}-x_{g3}}\left[\frac{x_{g1}\ln x_{g1}}{x_{g1}-1} - \frac{x_{g3}\ln x_{g3}}{x_{g3}-1}\right], \quad \text{etc.}$$

The partial lifetime of proton is calculable with order 1 coefficient.

$$\tau(p \rightarrow K^+ \bar{\nu})^{\rm exp} = 2 \times 10^{33} {\rm years}$$

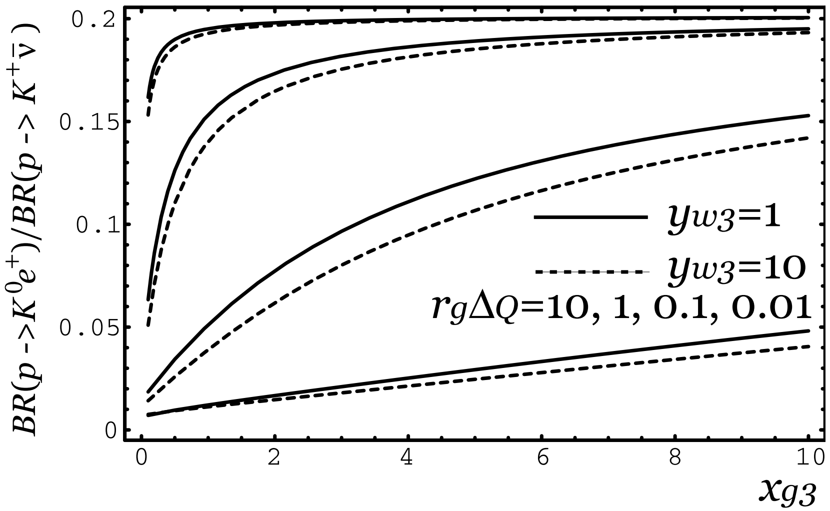
Shiozawa,NOON2003



Then, we can obtain the upper bound of the coefficient of the dim.5 operator.

$$C_L^{MSSM} < 10^{-(6\sim7)} \quad \Rightarrow \quad C_L^{Q6} < 10^{-(4\sim5)}$$

The relative decay rate induced by the same(LLLL) operator is also calculable .



$$\frac{BR(p\to K^0e^+)}{BR(p\to K^+\bar\nu)} = \left(3\times10^{-4}\sim0.2\right)$$

$$\Leftrightarrow\quad 2\times10^{-6}\ (\text{m -}SU(5))$$

$$\frac{BR(p\to K^0\mu^+)}{BR(p\to K^+\bar\nu)} = \left(7\times10^{-9}\sim5\times10^{-6}\right)$$

$$\Leftrightarrow\quad 7\times10^{-4}\ (\text{m}-SU(5))$$

$$\frac{B(p\to K^0(\pi^0)\mu^+)}{B(p\to K^0(\pi^0)e^+)} = \frac{|U_{eL}^{32}|^2}{|U_{eL}^{31}|^2} = \left(\frac{m_e}{m_\mu}\right)^2 \sim 2.4\times10^{-5}$$

$$\Leftrightarrow\quad \frac{B(p\to K^0\mu^+)}{B(p\to K^0e^+)} \sim \left(\frac{m_s}{m_u\sin\theta_C}\right)^2 \sim 10^3$$ (m-SU(5))

5. Conclusion

We have considered the extended model of the MSSM with Q6 flavor symmetry, and shown:

1. Q6 symmetry can work to suppress SUSY FCNC, and the conditions of the degree of degeneracy for the scalar masses have been obtained.

$$\Delta_Q < 10,\ \Delta_d < 10^{-1},\ \Delta_u < 10^{+3}$$

2. Proton decay induced by the Q6 symmetric dim.5 operators are suppressed than that of the MSSM.

$$C_L^{MSSM} < 10^{-(6\sim7)} \qquad C_L^{Q6} < 10^{-(4\sim5)}$$

3. The relative decay rates in Q6 model are specific.

$$\frac{B(p \to K^0(\pi^0)\mu^+)}{B(p \to K^0(\pi^0)e^+)} = \frac{|U_{eL}^{32}|^2}{|U_{eL}^{31}|^2} = \left(\frac{m_e}{m_\mu}\right)^2 \sim 2.4 \times 10^{-5}$$

	Exp. bound	Q_6 Model
$\|(\delta^e_{12})_{LL}\|$	$4.0 \times 10^{-5}\ \tilde{m}^2_{\tilde{\ell}}$	$4.9 \times 10^{-3} \Delta a^{\ell}_{L}$
$\|(\delta^e_{12})_{RR}\|$	$9 \times 10^{-4}\ \tilde{m}^2_{\tilde{\ell}}$	$8.4 \times 10^{-8} \Delta a^{e}_{R}$
$\|(\delta^e_{12})_{LR}\|$	$8.4 \times 10^{-7}\ \tilde{m}^2_{\tilde{\ell}}$	$5.1 \times 10^{-6} (\tilde{A}^e_{b'} - \tilde{A}^e_{c}) \tilde{m}^{-1}_{\tilde{\ell}}$
$\|(\delta^e_{13})_{LL}\|$	$2 \times 10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$	$1.7 \times 10^{-5} \Delta a^{\ell}_{L}$
$\|(\delta^e_{13})_{RR}\|$	$3 \times 10^{-1}\ \tilde{m}^2_{\tilde{\ell}}$	$5.9 \times 10^{-2} \Delta a^{e}_{R}$
$\|(\delta^e_{13})_{LR}\|$	$1.7 \times 10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$	$3.1 \times 10^{-7} (\tilde{A}^e_{b} - \tilde{A}^e_{b'}) \tilde{m}^{-1}_{\tilde{\ell}}$
$\|(\delta^e_{23})_{LL}\|$	$2 \times 10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$	$8.4 \times 10^{-8} \Delta a^{\ell}_{L}$
$\|(\delta^e_{23})_{RR}\|$	$3 \times 10^{-1}\ \tilde{m}^2_{\tilde{\ell}}$	$1.4 \times 10^{-6} \Delta a^{e}_{R}$
$\|(\delta^e_{23})_{LR}\|$	$1 \times 10^{-2}\ \tilde{m}^2_{\tilde{\ell}}$	$1.5 \times 10^{-9} (\tilde{A}^e_{b} - \tilde{A}^e_{b'}) \tilde{m}^{-1}_{\tilde{\ell}}$
$\|(\delta^e_{23})_{LL}(\delta^e_{13})_{LL}\|$	$1 \times 10^{-4}\ \tilde{m}^2_{\tilde{\ell}}$	$1.4 \times 10^{-12} (\Delta a^{\ell}_{L})^2$
$\|(\delta^e_{23})_{RR}(\delta^e_{13})_{RR}\|$	$9 \times 10^{-4}\ \tilde{m}^2_{\tilde{\ell}}$	$8.4 \times 10^{-8} (\Delta a^{e}_{R})^2$
$\|(\delta^e_{23})_{LL}(\delta^e_{13})_{RR}\|$	$2 \times 10^{-5}\ \tilde{m}^2_{\tilde{\ell}}$	$5.0 \times 10^{-9} \Delta a^{\ell}_{L} \Delta a^{e}_{R}$
$\|(\delta^e_{23})_{RR}(\delta^e_{13})_{LL}\|$	$2 \times 10^{-5}\ \tilde{m}^2_{\tilde{\ell}}$	$2.4 \times 10^{-11} \Delta a^{\ell}_{L} \Delta a^{e}_{R}$

The degeneracy conditions

$$\Delta a^{\ell}_{L} < 10^{-2}, \quad \tilde{A}^e_{b'} - \tilde{A}^e_{c} < 10^{-1}$$