μ - τ 対称性はニュートリノ混合に何を示唆するか？

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This talk is based on work with prof Yasue

はじめに

$$U_{PMNS}^T\, m_\nu\, U_{PMNS} = m_\nu^{diag}$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

$$\sin^2\theta_{12} = 0.314(1^{+0.18}_{-0.15})$$

$$\sin^2\theta_{23} = 0.44(1^{+0.41}_{-0.22})$$

$$\sin^2\theta_{13} = 0.9(^{+2.3}_{-0.9}) \times 10^{-2}$$

$$\Delta m^2_{\odot} = 7.92(1 \pm 0.09) \times 10^{-5} eV^2$$

$$\Delta m^2_{atm} = 2.4(1^{+0.21}_{-0.26}) \times 10^{-3} eV^2$$

振動実験から

$$\sin^2\theta_{13} \ll 1 \qquad \frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \ll 1$$

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naturalness *G. ‘t Hooft (‘80)*

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μ - τ 対称性

μ - τ 対称性からの帰結

微小な破れによって $\cdot \sin^2\theta_{13} \ll 1$ $\cdot \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$ を理解するために

$$\nu_\mu \leftrightarrow -\sigma\nu_\tau \quad (\sigma = \pm 1)$$

$$m_\nu = \begin{pmatrix} a & b & -\sigma b \\ b & d & e \\ -\sigma b & e & -d \end{pmatrix}$$

⇓ $U_{PMNS}^T \, m_\nu \, U_{PMNS} = m_\nu^{diag}$

$\sin\theta_{13} = 0$ 又は $\sin\theta_{12} = 0$

θ_{23} が最大

μ - τ 対称性の微小な破れ

$$m_\nu = \underbrace{\begin{pmatrix} a & b & -\sigma b \\ b & d & e \\ -\sigma b & e & -d \end{pmatrix}}_{\mu\text{-}\tau\ \text{対称}} + \epsilon \underbrace{\begin{pmatrix} 0 & b' & \sigma b' \\ b' & d' & 0 \\ \sigma b' & 0 & d' \end{pmatrix}}_{\mu\text{-}\tau\ \text{対称性の破れ}}$$

$$(\epsilon \sim 0)$$

μ - τ 対称性の微小な破れ

$$m_\nu = \begin{pmatrix} a & b & -\sigma b \\ b & d & e \\ -\sigma b & e & -d \end{pmatrix} + \epsilon \begin{pmatrix} 0 & b' & \sigma b' \\ b' & d' & 0 \\ \sigma b' & 0 & d' \end{pmatrix}$$

$$\Downarrow \quad U_{PMNS}^T \, m_\nu \, U_{PMNS} = m_\nu^{diag}$$

$$m_i = f(a, b, \ldots)$$

$$\theta_{ij} = h(a, b, \ldots)$$

ν 質量と混合角

$$X = \frac{(c_{23} + \sigma s_{23})b + \epsilon(c_{23} - \sigma s_{23})b'}{c_{13}}$$

$$m_1 = \cos^2\theta_{12}\lambda_1 + \sin^2\theta_{12}\lambda_2 - \sin 2\theta_{12} X$$

$$m_2 = \sin^2\theta_{12}\lambda_1 + \cos^2\theta_{12}\lambda_2 - \sin 2\theta_{12} X$$

$$m_3 = \frac{1}{2}\left(\lambda_3 + a + \frac{\lambda_3 - a}{\cos 2\theta_{13}}\right)$$

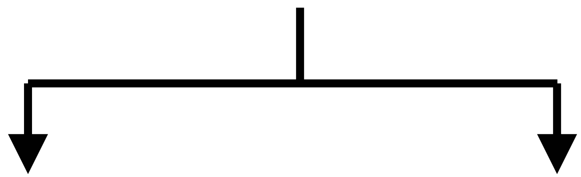
$$\vdots$$

実験結果を再現するために

$$\triangle m_{\odot}^2 = \frac{2\sqrt{2}(m_1 + m_2)X}{\sin 2\theta_{12}}$$

実験結果を再現するために

$$\Delta m_{\odot}^2 = \frac{2\sqrt{2}(m_1 + m_2)X}{\sin 2\theta_{12}} \ll \Delta m_{atm}^2$$



$X \sim 0$

$(m_1 + m_2) \sim 0$

ν 質量と混合角1

C1) $\sin\theta_{13} \to 0 (\epsilon \to 0)$

$$m_1 \approx \frac{a+d-\sigma e+O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$m_2 \approx \frac{a+d-\sigma e+O(\epsilon^2)}{2} + \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$m_3 \approx d+\sigma e+O(\epsilon^2)$$

$$\tan 2\theta_{12} \approx \frac{2X}{d-\sigma e-a+O(\epsilon^2)}$$

$$\tan 2\theta_{13} \approx \frac{2Y}{d+\sigma e-a+O(\epsilon^2)}$$

$$\cos 2\theta_{23} \approx 2\Delta$$

$$X \approx \sqrt{2}(b+O(\epsilon^2))$$

$$Y \approx \sqrt{2}\sigma(\epsilon b' - b\Delta)$$

$$\Delta \approx \frac{\sqrt{2}s_{13}b+\sigma\epsilon d'}{2e}$$

$\cdot \frac{\Delta m^2_\odot}{\Delta m^2_{atm}} \ll 1$ を得るために　（$\cdot X \sim 0$ の場合）

C1) $\sin\theta_{13} \to 0 (\epsilon \to 0)$

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$\cdot b \sim 0$　　$(d - \sigma e - a \sim 0)$

$\cdot \frac{\Delta m^2_\odot}{\Delta m^2_{atm}} \ll 1$ を得るために　（$\cdot m_1 + m_2 \sim 0$ の場合）

C1) $\sin\theta_{13} \to 0 (\epsilon \to 0)$

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$\cdot a + d - \sigma e \sim 0$

ν 質量と混合角2

C2) $\sin\theta_{12} \to O(\epsilon \to 0)$

$$m_1 \approx \frac{a+d+\sigma e+O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$m_2 \approx \frac{a+d+\sigma e+O(\epsilon^2)}{2} + \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$m_3 \approx d - \sigma e + O(\epsilon^2)$$

$$\tan 2\theta_{12} \approx \frac{2X}{d+\sigma e - a + O(\epsilon^2)}$$

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$$Y \approx -\sqrt{2}(b + O(\epsilon^2))$$

$$\Delta \approx \frac{\sigma\epsilon d' - \sqrt{2}s_{13}b}{2e+\sqrt{2}s_{13}b}$$

$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \sim X \ll 1$$

$\cdot \sin^2\theta_{13} \ll 1$ を得るために

C2）$\sin\theta_{12} \sim O(\epsilon)$

$$m_1 \approx \frac{a+d+\sigma e+O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

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$\cdot\, b \sim 0$　　（ $d+\sigma e-a \sim O(\epsilon)$ ）

まとめ

μ - τ 対称性を基調に $\sin^2\theta_{13} \ll 1$ $\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \ll 1$ の導出を考えた

C1). $\sin\theta_{13} \to 0(\epsilon \to 0)$ **の場合**

- $a + d + \sigma e \sim 0$ ($\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \ll 1$) 又は
  - $b \sim 0$ ($\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \ll 1$)
  - $d - \sigma e - a \sim 0$ ($\tan 2\theta_{12} \sim O(1)$)

C2). $\sin\theta_{12} \to 0(\epsilon \to 0)$ **の場合**

- $b \sim 0$ ($\sin^2\theta_{13} \ll 1$)
- $d + \sigma e - a \sim 0$ ($\tan 2\theta_{12} \sim O(1)$)

まとめ

C1). $\sin\theta_{13} \to 0(\epsilon \to 0)$ **の場合**

- $a + d + \sigma e \sim 0$ $\left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \ll 1\right)$ 又は

$$\begin{cases} \bullet\ b \sim 0 & \left(\frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \ll 1\right) \\ \bullet\ d - \sigma e - a \sim 0 & (\tan 2\theta_{12} \sim O(1)) \end{cases}$$

C2). $\sin\theta_{12} \to 0(\epsilon \to 0)$ **の場合**

$$\begin{cases} \bullet\ b \sim 0 & (\sin^2\theta_{13} \ll 1) \\ \bullet\ d + \sigma e - a \sim 0 & (\tan 2\theta_{12} \sim O(1)) \end{cases}$$

考察

C1). $\sin\theta_{13} \to 0 \,(\epsilon \to 0)$ **の場合**

μ - τ 対称性 $\Rightarrow$ $\sin^2\theta_{13} \ll 1$

L_e $\Rightarrow$ $\dfrac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \ll 1$

C2). $\sin\theta_{12} \to 0 \,(\epsilon \to 0)$ **の場合**

μ - τ 対称性 $\Rightarrow$ $\dfrac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \ll 1$

L_e $\Rightarrow$ $\sin^2\theta_{13} \ll 1$

ν 質量と混合角1

C1） $\sin\theta_{13} \sim O(\epsilon)$

$$m_1 \approx \frac{a+d-\sigma e+O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

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$$\Delta \approx \frac{\sqrt{2}s_{13}b+\sigma\epsilon d'}{2e}$$

$$\sin\theta_{13} \sim O(\epsilon) \sim \cos 2\theta_{23}$$

$\frac{\Delta m^2_\odot}{\Delta m^2_{atm}} \ll 1$ を得るために　（$X \sim 0$ の場合）

C1) $\sin\theta_{13} \to 0 (\epsilon \to 0)$

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$$\frac{\Delta m^2_\odot}{\Delta m^2_{atm}} \sim X \sim O(\epsilon^2) \sim \sin\theta_{13}^2$$

$\cdot \frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \ll 1$ を得るために　（$\cdot m_1 + m_2 \sim 0$ の場合）

C1) $\sin\theta_{13} \to 0 (\epsilon \to 0)$

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$$\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \sim m_1+m_2 \sim O(\epsilon^2) \sim \sin\theta_{13}^2$$

ν 質量と混合角2

C2） $\sin\theta_{12} \sim O(\epsilon)$

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$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \sim X \sim O(\epsilon) \sim \cos 2\theta_{23}$$

近似的 μ - τ 対称性から導かれる関係式

C1). $\sin\theta_{13} \to 0(\epsilon \to 0)$ **の場合**

$$\sin\theta_{13} \sim \cos 2\theta_{23} \qquad \left(\text{かつ } \frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \sim \sin\theta^2_{13}\right)$$

C2). $\sin\theta_{12} \to 0(\epsilon \to 0)$ **の場合**

$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \sim \cos 2\theta_{23}$$

実験結果を再現するために

$$b \sim 0$$

⇩

近似的に L_e が保存 (normal hierarchy)

⇩

$$X \approx \sqrt{2}(b + O(\epsilon^2)) \sim 0$$

$$\frac{\Delta m^2_{\odot}}{\Delta m^2_{atm}} \sim X \ll 1$$