複素ニュートリノ質量行列とPMNSユニタリー行列

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導入

未知のニュートリノ物理

↓

ニュートリノフレーバー質量行列: M_{flavor}

$$M_{\text{flavor}} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \text{ for } \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \Leftarrow M_{\text{charged}} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

↓

PMNSユニタリ行列: U_{PMNS}

↓

実験との比較

$$\Delta m_{atm}^2 = 2.4\left(1^{+0.21}_{-0.26}\right)\times 10^{-5}\ \text{eV}^2 \gg \Delta m_{\text{e}}^2 = 7.92(1\pm 0.09)\times 10^{-5}\ \text{eV}^2$$

$$\sin^2\theta_{12} = 0.314\left(1^{+0.18}_{-0.15}\right) : \sin^2\theta_{23} = 0.44\left(1^{+0.41}_{-0.22}\right) \gg \sin^2\theta_{13} = 0.9^{+2.3}_{-0.9}\times 10^{-2}$$

比較

未知のニュートリノ物理

現在、知っている形を導くように調節する

Normal mass hierarchy

$$m_0 \begin{pmatrix} :\ 0 & :\ \varepsilon & :\ -\sigma\varepsilon \\ :\ \varepsilon & :\ 1 & :\ \sigma \\ :\ -\sigma\varepsilon & :\ \sigma & :\ 1 \end{pmatrix}$$

Inveted mass hierarchy

$$m_0 \begin{pmatrix} :\ 2 & :\ \varepsilon & :\ -\sigma\varepsilon \\ :\ \varepsilon & :\ 1 & :\ -\sigma \\ :\ -\sigma\varepsilon & :\ -\sigma & :\ 1 \end{pmatrix}$$

$$(\sigma = \pm 1)$$

もっと一般的な制限をみつけたぁ～♫いぃ～♫

方法

ニュートリノフレーバー質量行列：M_{flavor}

PMNSユニタリ行列：U_{PMNS}

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\boldsymbol{M}_{\textbf{flavor}}$の固有ベクトル$\left|\lambda_{1,2,3}\right\rangle$

$$U_{\text{PMNS}} = \left(\left|\lambda_1\right\rangle, \left|\lambda_2\right\rangle, \left|\lambda_3\right\rangle\right)$$

複素行列の対角化

$$M_{\text{flavor}} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$

$$\boldsymbol{M} = M^{\dagger}_{\text{flavor}} M_{\text{flavor}} = \begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix}$$

$$A = |a|^2 + |b|^2 + |c|^2,\ B = a^* b + b^* d + c^* e,\ C = a^* c + b^* e + c^* f$$

$$D = |b|^2 + |d|^2 + |e|^2,\ E = b^* c + d^* e + e^* f,\ F = |c|^2 + |e|^2 + |f|^2$$

μ-τ対称性とその破れ

$$M_{\text{flavor}} = \underbrace{\begin{pmatrix} a & b_+ & -\sigma b_+ \\ b_+ & d_+ & e \\ -\sigma b_+ & e & d_+ \end{pmatrix}}_{\mu-\tau\text{対称}} + \underbrace{\begin{pmatrix} 0 & b_- & \sigma b_- \\ b_- & d_- & 0 \\ \sigma b_- & 0 & d_- \end{pmatrix}}_{\mu-\tau\text{非対称}}$$

$$\boldsymbol{M} = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & \mathrm{Re}(E) \\ -\sigma B_+^* & \mathrm{Re}(E) & D_+ \end{pmatrix} + \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-^* & D_- & i\,\mathrm{Im}(E) \\ \sigma B_-^* & -i\,\mathrm{Im}(E) & -D_- \end{pmatrix}$$

$$A = |a|^2 + 2\left(|b_+|^2 + |b_-|^2\right)$$

$$B_+ = a^* b_+ + b_+^*(d_+ - \sigma e) + b_-^* d_-,\quad B_- = a^* b_- + b_-^*(d_+ + \sigma e) + b_+^* d_-$$

$$D_+ = |b_+|^2 + |b_-|^2 + |d_+|^2 + |d_-|^2 + |e|^2,\quad D_- = 2\,\mathrm{Re}\left(b_-^* b_+ + d_-^* d_+\right)$$

$$\mathrm{Re}(E) = \sigma\left(|b_-|^2 - |b_+|^2\right) + 2\,\mathrm{Re}\left(d_+^* e\right),\quad \mathrm{Im}(E) = 2\,\mathrm{Im}\left(d_-^* e - \sigma b_-^* b_+\right)$$

μ-τ対称部の対角化

$$M_{\text{sym}} = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & \mathrm{Re}(E) \\ -\sigma B_+^* & \mathrm{Re}(E) & D_+ \end{pmatrix}$$

⇓

$$\boxed{B_+ = |B_+| e^{i\rho}}$$

⇓

$$\begin{pmatrix} A & |B_+| e^{i\rho} & -\sigma |B_+| e^{i\rho} \\ |B_+| e^{-i\rho} & D_+ & \mathrm{Re}(E) \\ -\sigma |B_+| e^{-i\rho} & \mathrm{Re}(E) & D_+ \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \text{ for } \lambda = D_+ + \sigma \mathrm{Re}(E)$$

残りの2つの固有ベクトル・・・

$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \sigma\\ 1\end{pmatrix}$$

直交性 ⇒

$$\frac{1}{\sqrt{2\left(|w|^2+|z|^2\right)}}\begin{pmatrix}\sqrt{2}w\\ -z\\ \sigma z\end{pmatrix} \quad \text{with } w, z = \text{complex}$$

$$\begin{pmatrix} A & |B_+|e^{i\rho} & -\sigma|B_+|e^{i\rho} \\ |B_+|e^{-i\rho} & D_+ & \mathrm{Re}(E) \\ -\sigma|B_+|e^{-i\rho} & \mathrm{Re}(E) & D_+ \end{pmatrix}\begin{pmatrix}\sqrt{2}w\\ -z\\ \sigma z\end{pmatrix} = \lambda\begin{pmatrix}\sqrt{2}w\\ -z\\ \sigma z\end{pmatrix}$$

$$(*)\quad \cos\theta = \frac{|w|}{\sqrt{|w|^2+|z|^2}}$$

$$\boxed{w = |w|,\ ze^{i\rho} = |z|}$$

⇓

$$\frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\cos\theta\\ -e^{-i\rho}\sin\theta\\ \sigma e^{-i\rho}\sin\theta\end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}e^{i\rho}\sin\theta\\ \cos\theta\\ -\sigma\cos\theta\end{pmatrix} \quad \text{for} \quad (A-\lambda)(D_+-\sigma E_+-\lambda) = 2|B_+|^2$$

μ-τ対称部のU_{PMNS}

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix},\ \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\cos\theta \\ -e^{-i\rho}\sin\theta \\ \sigma e^{-i\rho}\sin\theta \end{pmatrix},\ \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}e^{i\rho}\sin\theta \\ \cos\theta \\ -\sigma\cos\theta \end{pmatrix}$$

$$U_{PMNS} = \left(\frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\cos\theta \\ -e^{-i\rho}\sin\theta \\ \sigma e^{-i\rho}\sin\theta \end{pmatrix} \ \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}e^{i\rho}\sin\theta \\ \cos\theta \\ -\sigma\cos\theta \end{pmatrix} \ \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \right)$$

$$U_{PMNS} = \left(\frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\cos\theta \\ -e^{-i\rho}\sin\theta \\ \sigma e^{-i\rho}\sin\theta \end{pmatrix} \ \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \qquad\qquad \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}e^{i\rho}\sin\theta \\ \cos\theta \\ -\sigma\cos\theta \end{pmatrix} \right)$$

μ-τ対称部の混合角 I

$$U_{PMNS} = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\cos\theta \\ -e^{-i\rho}\sin\theta \\ \sigma e^{-i\rho}\sin\theta \end{pmatrix} \; \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}e^{i\rho}\sin\theta \\ \cos\theta \\ -\sigma\cos\theta \end{pmatrix} \; \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \right)$$

$$U_{PMNS} = \begin{pmatrix} c_{12} & s_{12}e^{i\rho} & 0 \\ -c_{23}s_{12}e^{-i\rho} & c_{23}c_{12} & s_{23} \\ s_{23}s_{12}e^{-i\rho} & -s_{23}c_{12} & c_{23} \end{pmatrix}$$

$$\theta_{12} = \theta, \ \cos\theta_{23} = \sigma\sin\theta_{23} = \frac{1}{\sqrt{2}}, \ \sin\theta_{13} = 0, \ B_0 = |B_0| e^{i\rho}$$

μ-τ対称部の混合角 II

$$U_{PMNS} = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\cos\theta \\ -e^{-i\rho}\sin\theta \\ \sigma e^{-i\rho}\sin\theta \end{pmatrix} \quad \frac{\sigma}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}e^{i\rho}\sin\theta \\ \cos\theta \\ -\sigma\cos\theta \end{pmatrix} \right)$$

⇩

$$U_{PMNS} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ s_{23}s_{13}e^{i\delta} & c_{23} & s_{23}c_{13} \\ c_{23}s_{13}e^{i\delta} & -s_{23} & c_{23}c_{13} \end{pmatrix}$$

⇩

$$\sin\theta_{12} = 0,\ \cos\theta_{23} = -\sigma\sin\theta_{23} = \frac{1}{\sqrt{2}},\ \theta_{13} = \theta,\ \rho = -\delta$$

μ-τ非対称部の影響

摂動による解析: ε

$$\theta_{12} = \theta,\ \cos\theta_{23} = \sigma \sin\theta_{23} = \frac{1}{\sqrt{2}},\ \sin\theta_{13} = 0,\ B_0 = |B_0| e^{i\rho}$$

↓

$$\cos 2\theta_{23} = O(\varepsilon),\ \sin\theta_{13} = O(\varepsilon)$$

○

$$\sin\theta_{12} = 0,\ \cos\theta_{23} = -\sigma \sin\theta_{23} = \frac{1}{\sqrt{2}},\ \theta_{13} = \theta,\ \rho = -\delta$$

↓

$$\cos 2\theta_{23} = O(\varepsilon),\ \sin\theta_{12} = O(\varepsilon)$$

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μ-τ非対称部の影響:摂動の1次

$$\left|1^{(1)}\right\rangle = -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}c_{12}B\_e^{-i\rho} - s_{12}\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_1^{(0)2}}\right)^* e^{-i\rho}\begin{pmatrix}0\\1\\\sigma\end{pmatrix}$$

$$\left|2^{(1)}\right\rangle = -\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}s_{12}B\_e^{-i\rho} + c_{12}\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_2^{(0)2}}\right)^* \begin{pmatrix}0\\1\\\sigma\end{pmatrix}$$

$m_{1,2,3}^{(0)2}$: treeでの質量

$m_3^{(0)2} = D_+ + \sigma\,\mathrm{Re}(E)$ 等

$$\left|3^{(1)}\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\sigma\left(\frac{\sqrt{2}c_{12}^2B\_e^{-i\rho} - c_{12}s_{12}\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_1^{(0)2}} + \frac{\sqrt{2}s_{12}^2B\_e^{-i\rho} + c_{12}s_{12}\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_2^{(0)2}}\right)e^{i\rho} \\ -\sigma\left(\frac{\sqrt{2}c_{12}s_{12}B\_e^{-i\rho} - s_{12}^2\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_1^{(0)2}} - \frac{\sqrt{2}c_{12}s_{12}B\_e^{-i\rho} + c_{12}^2\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_2^{(0)2}}\right) \\ \frac{\sqrt{2}c_{12}s_{12}B\_e^{-i\rho} - s_{12}^2\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_1^{(0)2}} - \frac{\sqrt{2}c_{12}s_{12}B\_e^{-i\rho} + c_{12}^2\left(D\_ + \sigma i\,\mathrm{Im}(E)\right)}{m_3^{(0)2} - m_2^{(0)2}}\end{pmatrix}$$

摂動の1次を含めたU_{PMNS} v.s. |3>

$$|3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 4\sigma \dfrac{\left(2\Delta m_{atm}^2 + \Delta m_e^2 \cos 2\theta_{12}\right)\boxed{B_-}e^{-i\rho} + \Delta m_e^2 \sin 2\theta_{12}\boxed{D_- + \sigma i\,\mathrm{Im}(E)}}{\left(2\Delta m_{atm}^2 - \Delta m_e^2\right)\left(2\Delta m_{atm}^2 + \Delta m_e^2\right)} e^{i\rho} \\ \sigma\left(1 + 2\dfrac{\left(2\Delta m_{atm}^2 - \Delta m_e^2 \cos 2\theta_{12}\right)\boxed{D_- + \sigma i\,\mathrm{Im}(E)} + \sqrt{2}\Delta m_e^2 \sin 2\theta_{12}\boxed{B_-}e^{-i\rho}}{\left(2\Delta m_{atm}^2 - \Delta m_e^2\right)\left(2\Delta m_{atm}^2 + \Delta m_e^2\right)}\right) \\ 1 - 2\dfrac{\left(2\Delta m_{atm}^2 - \Delta m_e^2 \cos 2\theta_{12}\right)\boxed{D_- + \sigma i\,\mathrm{Im}(E)} + \sqrt{2}\Delta m_e^2 \sin 2\theta_{12}\boxed{B_-}e^{-i\rho}}{\left(2\Delta m_{atm}^2 - \Delta m_e^2\right)\left(2\Delta m_{atm}^2 + \Delta m_e^2\right)} \end{pmatrix}$$

$$(*)\ \Delta m_e^2 = m_2^{(0)2} - m_1^{(0)2} > 0, \quad \Delta m_{atm}^2 = m_3^{(0)2} - \frac{m_1^{(0)2} + m_2^{(0)2}}{2}$$

$$c_{12}^2 m_1^{(0)2} + s_{12}^2 m_2^{(0)2} - m_3^{(0)2} = \frac{1}{2}\left(\Delta m_e^2 \cos 2\theta_{12} - 2\Delta m_{atm}^2\right)$$

$$s_{12}^2 m_1^{(0)2} + c_{12}^2 m_2^{(0)2} - m_3^{(0)2} = -\frac{1}{2}\left(\Delta m_e^2 \cos 2\theta_{12} + 2\Delta m_{atm}^2\right)$$

$$m_3^{(0)2} - m_2^{(0)2} = \frac{1}{2}\left(2\Delta m_{atm}^2 - \Delta m_e^2\right), \quad m_3^{(0)2} - m_1^{(0)2} = \frac{1}{2}\left(2\Delta m_{atm}^2 + \Delta m_e^2\right)$$

この｜3＞を要素として持つU_{PMNS}の形

$$|3\rangle = \begin{pmatrix} s_{13}e^{-i\delta} \\ s_{23}e^{i\gamma} \\ c_{23}e^{-i\gamma} \end{pmatrix} \approx \begin{pmatrix} s_{13}e^{-i\delta} \\ \sigma\dfrac{1-\Delta+i\gamma}{\sqrt{2}} \\ \dfrac{1+\Delta-i\gamma}{\sqrt{2}} \end{pmatrix}$$ with $|\Delta| = O(\varepsilon)$ & $|\gamma| = O(\varepsilon)$

$$s_{13}e^{-i(\rho+\delta)} \approx \frac{\sqrt{2}\sigma e^{-i\rho}\boldsymbol{B}_-}{\Delta m^2_{atm}}$$

$$\Rightarrow \boxed{\boldsymbol{B}_- \approx \pm|\boldsymbol{B}_-|e^{-i\delta}}$$

｜1＞, ｜2＞も同様に・・・

$$s_{13}e^{-i(\rho+\delta)} \approx \sigma\frac{2\sqrt{2}e^{-i\rho}\boldsymbol{B}_- + \boldsymbol{R}\sin 2\theta_{12}\left(D_- + \sigma i\,\mathrm{Im}(E)\right)}{2\Delta m^2_{atm}}$$

$$\boldsymbol{R} = \frac{\Delta m^2_e}{\Delta m^2_{atm}} << 1$$

$$\Delta \approx -\frac{2D_- + \sigma s_{13}\cos(\rho+\delta)\sin 2\theta_{12}\Delta m^2_e}{2\Delta m^2_{atm}}$$

$$J = -c_{23}c_{12}s_{13}s_{23}c_{13}s_{12}c_{13}s_{\rho+\delta} \Rightarrow \rho+\delta$$

$$\gamma \approx \frac{2\sigma\,\mathrm{Im}(E) - \sigma s_{13}\sin(\rho+\delta)\sin 2\theta_{12}\Delta m^2_e}{2\Delta m^2_{atm}}$$

U_{PMNS}への示唆

摂動範囲を超えて・・・U_{PMNS}の形

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} e^{i\rho} \\ -\sin\theta_{12} e^{-i\rho} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} e^{i\gamma} & 0 \\ 0 & e^{-i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta_{13} & \sin\theta_{13} e^{-i\delta} \\ -\sin\theta_{13} e^{i\delta} & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_3 \end{pmatrix}$$

$(*)$ Majorana phases for $\nu_{1,2,3}$

U_{PMNS}からの示唆

$$\begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix} \text{ with } U_{PMNS}(\rho,\gamma,\delta)$$

$$\begin{pmatrix} A & e^{i(\gamma-\rho)}B & e^{-i(\gamma+\rho)}C \\ e^{-i(\gamma-\rho)}B^* & D & e^{-2i\gamma}E \\ e^{i(\gamma+\rho)}C^* & e^{2i\gamma}E^* & F \end{pmatrix} \text{ with } U_{PMNS}^{PDG}(\rho+\delta)$$

$$\begin{pmatrix} e^{2i\rho}a & e^{i(\gamma+\rho)}b & e^{-i(\gamma-\rho)}c \\ e^{i(\gamma+\rho)}b & e^{2i\gamma}d & e \\ e^{-i(\gamma-\rho)}c & e & e^{-2i\gamma}f \end{pmatrix} \text{ with } U_{PMNS}^{PDG}(\rho+\delta)$$

応用

Mohapatra & Rodejohann, PRD **72** ('05)

$$\begin{pmatrix} 0 & \beta\varepsilon & -\sigma\beta\varepsilon e^{i\alpha} \\ \beta\varepsilon & 1+\varepsilon & \sigma \\ -\sigma\beta\varepsilon e^{i\alpha} & \sigma & 1+\varepsilon \end{pmatrix}$$

$$\boldsymbol{a}=0,\ \boldsymbol{b}_+=\frac{1+e^{i\alpha}}{2}\beta\varepsilon$$

$$\boldsymbol{d}_+=1+\varepsilon$$

$$\boldsymbol{\sigma e}=1 \qquad \boldsymbol{d}_-=0$$

$$\begin{pmatrix} A & e^{i(\gamma-\rho)}B & e^{-i(\gamma+\rho)}C \\ e^{-i(\gamma-\rho)}B^* & D & e^{-2i\gamma}E \\ e^{i(\gamma+\rho)}C^* & e^{2i\gamma}E^* & F \end{pmatrix} \text{ with } U^{PDG}_{PMNS}(\rho+\delta)$$

$$\boxed{B_+=|B_+|e^{i\rho}} = 0b_+ + b_+^*(d_+-\sigma e)+b_-^*0 = \frac{1+e^{-i\alpha}}{2}\beta\varepsilon^2 = e^{-i\alpha/2}\beta\varepsilon^2\cos\left(\frac{\alpha}{2}\right)$$

$$\boxed{B_-=\pm|B_+|e^{-i\delta}} = a^*b_- + b_-^*(d_++\sigma e)+b_+^*d_- = \frac{1-e^{-i\alpha}}{2}2\beta\varepsilon = 2ie^{-i\alpha/2}\beta\varepsilon\sin\left(\frac{\alpha}{2}\right)$$

$$\rho=-\frac{\alpha}{2} \qquad -\delta=\frac{\pi}{2}-\frac{\alpha}{2}$$

$$\rho \approx -\frac{\alpha}{2},\ \gamma \approx 0,\ \delta \approx \frac{\alpha-\pi}{2} \Rightarrow \rho+\delta \approx -\frac{\pi}{2}$$

$$\begin{pmatrix} e^{2i\rho}a & e^{i(\gamma+\rho)}b & e^{-i(\gamma-\rho)}c \\ e^{i(\gamma+\rho)}b & e^{2i\gamma}d & e \\ e^{-i(\gamma-\rho)}c & e & e^{-2i\gamma}f \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & e^{-i\alpha/2}\beta\varepsilon & -e^{-i\alpha/2}\sigma\beta\varepsilon e^{i\alpha} \\ e^{-i\alpha/2}\beta\varepsilon & 1+\varepsilon & \sigma \\ -e^{-i\alpha/2}\sigma\beta\varepsilon e^{i\alpha} & \sigma & 1+\varepsilon \end{pmatrix}$$

with $U_{PMNS}^{PDG}(\rho+\delta)$

$$z = e^{-i\alpha/2}\beta\varepsilon$$

$$\begin{pmatrix} 0 & z & -\sigma z^* \\ z & 1+\varepsilon & \sigma \\ -\sigma z^* & \sigma & 1+\varepsilon \end{pmatrix}$$

αに依らずに(ほぼ)最大のCP の破れ with $U_{PMNS}^{PDG}(\rho+\delta)$

まとめ

$$\begin{pmatrix} e^{2i\rho}a & e^{i(\gamma+\rho)}b & e^{-i(\gamma-\rho)}c \\ e^{i(\gamma+\rho)}b & e^{2i\gamma}d & e \\ e^{-i(\gamma-\rho)}c & e & e^{-2i\gamma}f \end{pmatrix} \Leftrightarrow \begin{pmatrix} A & e^{i(\gamma-\rho)}B & e^{-i(\gamma+\rho)}C \\ e^{-i(\gamma-\rho)}B^* & D & e^{-2i\gamma}E \\ e^{i(\gamma+\rho)}C^* & e^{2i\gamma}E^* & F \end{pmatrix}$$

with $U_{PMNS}^{PDG}(\rho+\delta)$

$$\boldsymbol{B}_+ \approx |\boldsymbol{B}_+| e^{i\rho} \qquad \boldsymbol{B}_- \approx \pm |\boldsymbol{B}_-| e^{-i\delta} \quad \text{for } \boldsymbol{R} \approx 0$$

$$s_{13}e^{-i(\rho+\delta)} \approx \sigma \frac{2\sqrt{2}e^{-i\rho}\boldsymbol{B}_- + \boldsymbol{R}\sin 2\theta_{12}\left(D_- + \sigma i \,\mathrm{Im}(E)\right)}{2\Delta m_{atm}^2}$$

$$\Delta \approx -\frac{2D_- + \sigma s_{13}\cos(\rho+\delta)\sin 2\theta_{12}\Delta m_{\mathrm{e}}^2}{2\Delta m_{atm}^2}$$

$$\gamma \approx \frac{2\sigma\,\mathrm{Im}(E) - \sigma s_{13}\sin(\rho+\delta)\sin 2\theta_{12}\Delta m_{\mathrm{e}}^2}{2\Delta m_{atm}^2}$$

$$\boldsymbol{R} = \frac{\Delta m_{\mathrm{e}}^2}{\Delta m_{atm}^2} << 1$$