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# **Empirical Neutrino Mass Matrix Related to Up-Quark Masses**

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# Abstract

Based on a new approach to quark and lepton masses, a neutrino mass matrix of a new type is speculated. The mass matrix is described in terms of the up-quark and charged lepton masses, and it can lead to a nearly tribimaximal mixing without assuming any discrete symmetry.

**1 Phenomenological Aspect**

**2 New approach to quark and lepton masses and mixings**

**3 Remarks**

# 1 Phenomenological Aspect

- ◆ Present neutrino oscillation data favor to the so-called **tribimaximal mixing**

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- ◆ Models with such a mixing have usually been derived based on a discrete symmetry.
- ◆ Against this conventional anticipation, I found that **a neutrino mass matrix which is described in terms of up-quark and charged lepton masses** can beautifully lead to the tribimaximal mixing without assuming any discrete symmetry.

**Our neutrino mass matrix has very peculiar form:**

$$M_\nu = m_D M_R^{-1} m_D^T$$

**Dirac neutrino mass matrix  $m_D$  is given by**

$$m_D = M_e$$

**Majorana mass matrix  $M_R$  of  $\nu_R$  is given by**

$$M_R \propto M_e M_u^{1/2} + M_u^{1/2} M_e$$

**$M_e$ : the charged lepton mass matrix whose diagonal form is**

$$\langle M_e \rangle_e = (M_e)^D \equiv \text{diag}(m_e, m_\mu, m_\tau)$$

**$M_u$ : the up-quark mass matrix, and the diagonal form of  $M_u^{1/2}$  is**

$$\langle M_u^{1/2} \rangle_u \equiv (M_u^{1/2})^D \equiv \text{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$$

**When  $M_f$  takes a diagonal form on a flavor basis, we will call the basis " **$f$ -basis**", and denote a matrix  $A$  on the  $f$ -basis as  $\langle A \rangle_f$ .**

In order to obtain the neutrino mixing matrix, we must calculate the mass matrix on the  $e$ -basis:

$$\langle M_\nu \rangle_e = k \langle M_e \rangle_e \left[ \langle M_u^{1/2} \rangle_e \langle M_e \rangle_e + \langle M_e \rangle_e \langle M_u^{1/2} \rangle_e \right]^{-1} \langle M_e \rangle_e$$

In order to calculate  $\langle M_\nu \rangle_e$ , we must know the form of  $\langle M_u^{1/2} \rangle_e$  on the  $e$ -basis. We know that the  $d$ -basis is connected to the  $u$ -basis by the CKM mixing matrix  $V$ :

How about the  $e$ -basis to the  $u$ -basis?

At present, we do not know such a theoretical relation.

We assume the form

$$\langle M_u^{1/2} \rangle_e = V^T(\delta_{ue}) \langle M_u^{1/2} \rangle_u V(\delta_{ue})$$

on the analogy of  $\langle M_u \rangle_d = V^T(\delta) \langle M_u \rangle_u V(\delta)$

where  $V(\delta)$  is the standard expression of the CKM matrix.

By using the observed values of  $M_u^D$ ,  $M_e^D$ , and  $V(\delta)$ , we obtain:

# Numerical results

$\delta$	$\sin^2 2\theta_{23}$	$\tan^2 \theta_{12}$	$ U_{13} $	$\Delta m_{21}^2 / \Delta m_{32}^2$
0	0.3831	0.4170	0.01132	0.00262
60°	0.7574	0.4186	0.00919	0.00171
90°	0.9179	0.4469	0.00648	0.00118
120°	0.9818	0.4813	0.00386	0.00091
180°	0.9997	0.5125	0.00010	0.00074

Present data on the CKM matrix show  $\delta \simeq \pi/3$ .

The predicted value of  $\sin^2 2\theta_{23}$  at  $\delta = \pi/3$  is in poor agreement with the observed value  $\sin^2 2\theta_{23} \simeq 1$ .

However, we find that **the case with  $\delta_{ue} = \pi$  can beautifully leads to a tribimaximal mixing without assuming any discrete symmetry.** Do you think this is accidental?

We consider that this result is very suggestive.

Anyhow, this phenomenological discovery is the first evidence which shows a connection between the Yukawa coupling constants  $Y_f$  in the quark and lepton sectors.

## 2 New approach to quark and lepton masses and mixings

### ◆ Basic assumption

The origin of the mass spectra is due to the VEV structures of gauge singlet scalars  $(Y_f)_{ij}$

**Conventional**

$$M_f = (Y_f)_{ij} \langle H^0 \rangle$$

$(Y_f)_{ij}$  : coupling constants

We search constraints on  $Y_f$  under a flavor symmetry

**New approach**

$$M_f = \frac{1}{\Lambda} \langle (Y_f)_{ij} \rangle \langle H^0 \rangle$$

$(Y_f)_{ij}$  : scalar fields

We search VEV structures of  $Y_f$  from a given superpotential

## U(3) model

$Y_f$ : 1+8 of U(3)

$$Y_f^\dagger = Y_f$$

## O(3) model

1+5 of O(3)

$$Y_f^T = Y_f$$

Diagonalization

$$U_q^\dagger \langle Y_f \rangle U_q = \langle Y_q \rangle^D$$

$$U_e^T \langle Y_e \rangle U_e^* = \langle Y_e \rangle^D$$

$$U_f^T \langle Y_f \rangle U_f = \langle Y_f \rangle^D$$

References: Y.K. PLB662 (2008) 43

Y.K., arXiv:0804.4267

Y.K., arXiv:0803.3101

Hereafter, we adopt the O(3) model

$$W_Y = \sum_{i,j} \frac{y_u}{\Lambda} U_i(Y_u)_{ij} Q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} D_i(Y_d)_{ij} Q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} L_i(Y_\nu)_{ij} N_j H_u + \sum_{i,j} \frac{y_e}{\Lambda} L_i(Y_e)_{ij} E_j H_d + h.c. + \sum_{i,j} y_R N_i(Y_R)_{ij} N_j$$

## ◆ Prescription

(1) We give a superpotential  $W(Y_f, Y_R)$

(2) From the SUSY vacuum conditions

$$\frac{\partial W}{\partial Y_f} = 0$$

and so on,

we obtain relations among  $Y_f$

## ◆ Example

$$W_u = \lambda_u \text{Tr}[\Phi_u \Phi_u X_u] + m_u \text{Tr}[Y_u X_u] + W_{\Phi_u}(\Phi_u)$$

$$\frac{\partial W}{\partial Y_u} = 0 = m_u X_u$$

→

$$\langle X_u \rangle = 0$$

$$\frac{\partial W}{\partial X_u} = 0 = \lambda_u \Phi_u \Phi_u + m_u Y_u$$

→

$$\langle Y_u \rangle = -\frac{\lambda_u}{m_u} \langle \Phi_u \rangle \langle \Phi_u \rangle$$

Therefore, the operator  $Y_u^{1/2}$  corresponds to  $\Phi_u$

We assume a similar structure for  $W_e$

◆ Dirac mass matrix  $Y_\nu$

$$W_\nu = \lambda_\nu \phi_\nu \text{Tr}[Y_\nu X_\nu] + \lambda_{\nu e} \phi_e \text{Tr}[Y_e X_\nu]$$

$$\frac{\partial W}{\partial X_\nu} = 0 \quad \longrightarrow \quad Y_\nu = -\frac{\lambda_{\nu e} \phi_e}{\lambda_\nu \phi_\nu} Y_e$$

◆ Majorana mass matrix  $Y_R$

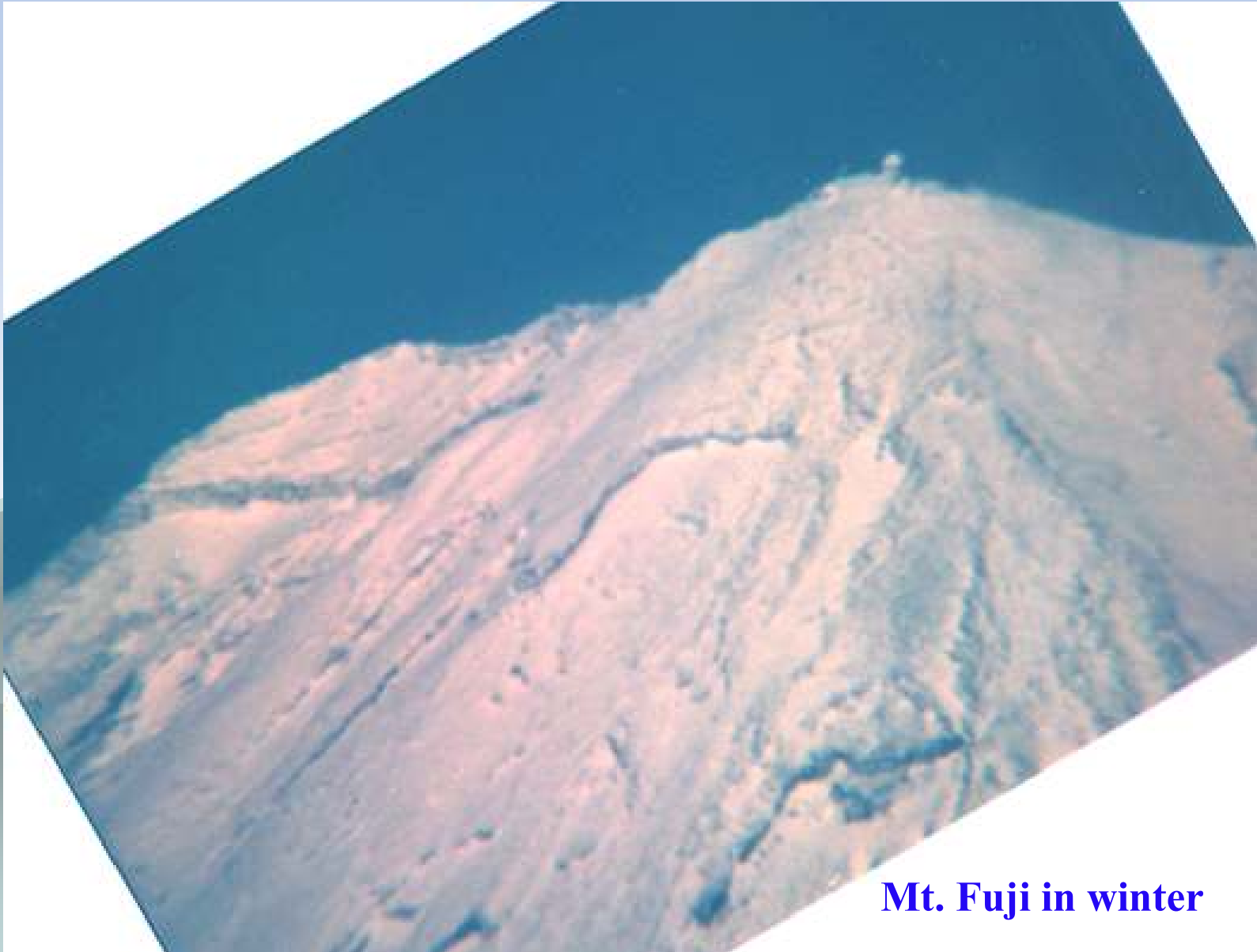
$$W_R = \lambda_R \text{Tr}[(Y_e \Phi_u + \Phi_u Y_e) X_R] + m_R \text{Tr}[Y_R X_R]$$

$$\frac{\partial W}{\partial X_R} = 0 \quad \longrightarrow \quad Y_R = -\frac{\lambda_R}{m_R} (Y_e \Phi_u + \Phi_u Y_e)$$

◆ Therefore, we obtain the neutrino mass matrix

$$\langle M_\nu \rangle_e = k \langle Y_e \rangle^D \left[ \langle Y_u^{1/2} \rangle_e \langle Y_e \rangle^D + \langle Y_e \rangle^D \langle Y_u^{1/2} \rangle_e \right]^{-1} (\langle Y_e \rangle^D)^T$$

# 3 Remarks



**Mt. Fuji in winter**

## Note on the O(3) model

Under the O(3) flavor symmetry, the fields  $\mathbf{3}$  and  $1+5=6$  are transformed as  $\mathbf{3} \rightarrow R \cdot \mathbf{3}$ ,  $\mathbf{6} \rightarrow R \cdot \mathbf{6} \cdot R^T$  where  $R$  is an orthogonal matrix. Our relations which are derived from the O(3) invariant superpotential  $W$  are valid only on bases which are related each other by orthogonal transformations.

On the other hand, the effective Yukawa coupling constants  $\langle Y_f \rangle$  in the O(3) model are diagonalized as  $U_f^T \langle Y_f \rangle U_f = \langle Y_f \rangle^D$  where  $U_f$  is a unitary matrix. Therefore, we assume that  $\Phi_u$  and  $\Phi_e$  (so that  $Y_u$  and  $Y_e$ ) are real, so that our relations are valid only on the  $u$ - and  $e$ -bases. Note that we cannot use our relations on the  $d$ -basis. The matrix  $\langle Y_d \rangle_d$  is obtained by diagonalizing the form  $\langle Y_d \rangle_u$  on the  $u$ -basis as  $V^T(\delta) \langle Y_d \rangle_u V(\delta) = \langle Y_d \rangle_d$

## Another byproducts in this approach

By assuming the form  $W_{\Phi_e}(\Phi_e)$  suitably, we can obtain a cubic equation

$$c_3^e \Phi_e \Phi_e \Phi_e + c_2^e \Phi_e \Phi_e + c_1^e \Phi_e + c_0^e 1 = 0,$$

so that we obtain relations

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{\text{Tr}[\Phi_e \Phi_e]}{\text{Tr}^2[\Phi_e]} = 1 - 2 \frac{c_1^e c_3^e}{(c_2^e)^2}$$

$$\frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{\det \Phi_e}{\text{Tr}^3[\Phi_e]} = \frac{c_0^e (c_3^e)^2}{(c_2^e)^3}$$

The observed values are  $2/3$  and  $1/(2 \cdot 3^5)$ , respectively. For a toy model which give those values, see YK, PL B662 (2008) 43.

# Prospects

In the present work, we have found an empirical neutrino mass matrix which leads to a nearly tribimaximal mixing without assuming any discrete symmetry, but this does not mean that a discrete symmetry is not applicable to the neutrino sector. Rather, we consider that this means that the discrete symmetry is applicable not only to the lepton sector, but also to the quark sector.

**This approach will shed a new light on the quark and lepton masses and mixings.**