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Lepton Masses based on a Broken U(3)

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based on YK, JHEP 08 (2007) 086 (hep-ph/0705.2275)
N.Haba and YK, hep-ph/0708.3913

1 Introduction

Lepton sector is a treasure house of clues to unification model of quarks and leptons

1.1 Motivation

[1] How do we understand an empirical charged lepton mass formula

The observed charged lepton masses satisfy

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \qquad (1.1)$$

with remarkable precession.

YK, Lett.Nuov.Cim. 342 (1982) 201; PLB 120B, (1983) 161

By inputting the observed values of m_e, m_μ ,

the formula (1.1) predicts the tau lepton mass

$$m_\tau = 1776.97 \quad \text{MeV} \qquad (1.2)$$

which is in en excellent agreement with the observed value

$$m_\tau^{exp} = 1776.99^{+0.29}_{-0.26} \quad \text{MeV} \qquad (1.3)$$

Note that it was 10 years after the prediction (1982) that the new value of tau mass was reported by ARGUS, BES and CLEO (1992)

Permutation symmetry S_3

We define the doublet (ϕ_π, ϕ_η) and singlet ϕ_σ of S_3:

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \quad (1.4)$$

from the three objects (ϕ_1, ϕ_2, ϕ_3)

Short review of a scalar potential model (1990)

(i) We assume an S_3 invariant potential

$$V(\phi) = m^2(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)$$

$$+\lambda_1(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)^2 + \lambda_2\phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2) \quad (1.5)$$

(ii) Then the minimizing condition of V leads to

$$v_\pi^2 + v_\eta^2 = v_\sigma^2 \quad (1.6)$$

which means the VEV relation

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2 . \quad (1.7)$$

However, the potential (1.5) is not a general form of S_3 invariant potential. We need a further constraint in addition to the S_3 symmetry.

[2] Ho do we build a model which gives a bilinear form $m_i \propto v_i^2$

1990 **Seesaw-type model**

$$L = y\bar{e}_{Li}\langle H_{Lik}\rangle M_E^{-1}\langle H_{Rkj}\rangle e_{Rj}$$

YK, MPL (1990)

However, the model induces a FCNC problem.

2007 **Froggatt-Nielsen-type model**

$$L = y\bar{\ell}_{Li}\left(\frac{\langle\Phi\rangle}{M}\right)^2_{ij} H_d e_{Rj}$$

YK, JHEP (2007)

However, the model includes a higher dimensional term

1.2 Two problems which we should solve

[1] How do we derive the VEV relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2 \quad (1.10)$$

reasonably?

[2] How do we make a model which gives

$$m_{ei} \propto v_i^2 \quad (1.11)$$

naturally?

2
How to get the VEV relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2$$

YK, JHEP 08 (2007) 086 [hep-ph/0705.22759]

Since we consider that quarks and leptons are triplets under a symmetry, it is natural to consider that the scalar ϕ is a nonet (3x3) of the symmetry.

We consider S_4 which is embedded into SU(3)

	SU(3)	S4	
ψ	1	1	ϕ_σ
ψ_i	3	3'	$\ell_{Li} = (\nu_i, e_i)_L,\ e_{Ri}$
$\psi_{(ij)}$	6	1+2+3	
ψ_i^j	8	2+3+3'	(ϕ_π, ϕ_η)
$\psi_{(ijk)}$	10	1'+3+3'+3'	

We assume that 1+2 of S_4 originate in 1+8 of SU(3)

Basic assumption

The fields ϕ_d and ϕ_u always appears in the theory in terms of the nonet form

$$\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \tag{2.1}$$

$$\begin{aligned} \phi_{11} &= \frac{1}{\sqrt{3}}\phi_\sigma + \frac{2}{\sqrt{6}}\phi_\eta \\ \phi_{22} &= \frac{1}{\sqrt{3}}\phi_\sigma - \frac{1}{\sqrt{6}}\phi_\eta - \frac{1}{\sqrt{2}}\phi_\pi \\ \phi_{33} &= \frac{1}{\sqrt{3}}\phi_\sigma - \frac{1}{\sqrt{6}}\phi_\eta + \frac{1}{\sqrt{2}}\phi_\pi \end{aligned} \tag{2.2}$$

We assume the following U(3) invariant superpotential

$$W = m\mathrm{Tr}(\Phi\Phi) + \lambda\mathrm{Tr}(\Phi\Phi\Phi) \qquad (2.3)$$

However, the condition $\partial W/\partial\Phi = 0$ cannot lead to a desirable VEV relation.

We assume a Z_2 parity for

$$\Phi = \phi^{(8)} + \phi^{(1)} \qquad (2.4)$$

- +

Tr[(9)(9)] = Tr[(8)(8) + (1)(1)] : no effect of Z_2

- - + +

Tr[(9)(9)(9)]=Tr[(8)(8)(8) + 3(1)(8)(8) +(1)(1)(1)]

- - - + - - + + +

forbidden

By using $\Phi^{(1)} = \frac{1}{3}\mathrm{Tr}\Phi$ and $\Phi^{(8)} = \Phi - \frac{1}{3}\mathrm{Tr}\Phi$,

we can rewrite

$$\begin{aligned} \mathrm{Tr}(\Phi\Phi\Phi) &\Rightarrow \mathrm{Tr}(3\Phi^{(1)}\Phi^{(8)}\Phi^{(8)}+\Phi^{(1)}\Phi^{(1)}\Phi^{(1)}) \\ &= \mathrm{Tr}(\Phi)\left[\mathrm{Tr}(\Phi\Phi) - \frac{2}{9}(\mathrm{Tr}\Phi)^2\right] \end{aligned} \qquad (2.5)$$

From the modified W under Z_2, we obtain

$$\frac{\partial W}{\partial \Phi} = 0 = f_1 \Phi + f_0 \mathbf{1} \tag{2.6}$$

$$f_1 = 2(m + \lambda \mathrm{Tr}\Phi) \tag{2.7}$$

$$f_0 = \lambda \left[\mathrm{Tr}(\Phi\Phi) - \frac{2}{3}(\mathrm{Tr}\Phi)^2 \right] \tag{2.8}$$

In order to obtain non-zero and non-degenerate eigenvalues of $\Phi \neq 0$, the coefficients $f_1=0$ and $f_0=0$ are required.

The requirement $f_0=0$ just means

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2 \tag{2.9}$$

for the diagonal basis of $\langle \Phi \rangle = \mathrm{diag}(v_1, v_2, v_3)$

3
How to Get the bilinear form

$$m_{ei} \propto v_i^2$$

N.Haba and YK, hep-ph/0708.3913

3.1 Basic assumptions

The basic concept is somewhat different form the previous scenario

(i) The symmetry U(3) is badly broken at the beginning,

not only in the Yukawa sector $Y_{ij} L_j H_d E_i$,

but also in the Φ sector.

(ii) Y is a common symmetry breaking parameter in the Yukawa and Φ sectors

(iii) Φ sector is only broken via a tadpole term

$$W = W_Y + W_\Phi \quad (3.1)$$

$$W_Y = Y_{ij}(LH_dE)_{ji} \quad (3.2)$$

$$W_\Phi = \lambda \mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] + m\mathrm{Tr}[\Phi\Phi] - \mu^2 \mathrm{Tr}[Y\Phi] \quad (3.3)$$

Note: Here, we do not assume the Z_2 symmetry as in the previous section. In the present section, the form of the cubic term is only an ad hoc assumption.

Also note that the cubic term is rewritten as

$$\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] = \mathrm{Tr}[\Phi\Phi\Phi] - \mathrm{Tr}[\Phi]\left(\mathrm{Tr}[\Phi\Phi] - \frac{2}{9}(\mathrm{Tr}[\Phi])^2\right) \quad (3.4)$$

by using the definition $\Phi^{(8)} = \Phi - \frac{1}{3}\mathrm{Tr}\Phi$

3.2 Vacuum condition

We obtain the condition

$$\frac{\partial W}{\partial \Phi} = 0 = 3\lambda\Phi\Phi + f_1(\Phi)\Phi$$

$$+f_0(\Phi)1 - \mu^2 Y \quad (3.5)$$

where

$$f_1(\Phi) = 2(m - \lambda \mathrm{Tr}[\Phi]), \quad (3.6)$$

$$f_0(\Phi) = -\lambda\left(\mathrm{Tr}[\Phi\Phi] - \frac{2}{3}(\mathrm{Tr}[\Phi])^2\right) \quad (3.7)$$

We can choose the solution which provides the bilinear relation

$$Y_{ij} = \frac{3\lambda}{\mu^2} \sum_k \langle \Phi_{ik} \rangle \langle \Phi_{kj} \rangle \tag{3.8}$$

together with

$$f_1(\Phi)\Phi + f_0(\Phi)1 = 0 \tag{3.9}$$

from Eq.(3.5).

In order to get the non-zero and non-degenerate eigenvalues of $\langle \Phi \rangle$, we must take f_1=0 and f_0=0. The condition f_0=0 just means the VEV relation (2.9).

3.3 Neutrino mass spectrum

Since $\langle \Phi \rangle$ is always expressed into a diagonal form by changing the flavor basis, mass relation is given by (1.6) and (1.7). We consider that the present scenario is also applicable to the neutrino sector.

Inputs: $R = \Delta m_{21}^2 / \Delta m_{32}^2 = 0.029$ (3.10)

$$m_{\nu 3} = \sqrt{\Delta m_{atm}^2} = \sqrt{0.00274} \text{ eV} \quad (3.11)$$

Seesaw masses

$$m_{\nu i} \propto v_i^4$$

$$m_{\nu 1} = 4.4 \times 10^{-5} \text{eV}$$
$$m_{\nu 2} = 8.7 \times 10^{-3} \text{eV}$$
$$m_{\nu 3} = 5.2 \times 10^{-2} \text{eV}$$

Dirac masses

$$m_{\nu i} \propto v_i^2$$

$$m_{\nu 1} = 3.5 \times 10^{-4} \text{eV}$$
$$m_{\nu 2} = 8.8 \times 10^{-3} \text{eV}$$
$$m_{\nu 3} = 5.2 \times 10^{-2} \text{eV}$$

3.4 Comments on the model in this section

(1) It may be that the choice (3.8) is fantastic (even crazy). However, as far as we adhere the non-zero and non-degenerate eigenvalues of $\langle \Phi \rangle$ the choice (3.8) is unique.

(2) In the present section, since we have investigated a model without a higher dimensional term, we regards the Yukawa coupling constant Y as the symmetry breaking parameter in the theory.

However, we can consider another scenario where Y is a VEV matrix of a field. The search of this possibility is our future task.

4
Concluding Remarks

Mass spectrum originates in the VEV structure of a nonet scalar Φ, where 9 of U(3) is broken into 1+2+3+3' of S_4

The both models in Secs.2 and 3 are based on very similar idea, but they are crucially different from each other on some points:

Model	Sec.2	Sec.3
Purpose	To get the VEV relation	To get the bilinear form
Yukawa c.c.	y; flavor-indepnd.	Y_{ji}: flavor-dependent
Yukawa int.	$y\bar{\ell}_{Li}\left(\frac{\Phi}{M}\right)^2_{ij} H_d e_{Rj}$	$Y_{ij}(LH_dE)_{ji}$
	Higher dimensional	Normal dimension
Symmetry	U(3) invariant	badly broken by Y
Cubic term in W	$\mathrm{Tr}(\Phi\Phi\Phi) \Rightarrow \mathrm{Tr}(\Phi)\left[\mathrm{Tr}(\Phi\Phi) - \frac{2}{9}(\mathrm{Tr}\Phi)^2\right]$	$\mathrm{Tr}[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] = \mathrm{Tr}[\Phi\Phi\Phi] - \mathrm{Tr}[\Phi]\left(\mathrm{Tr}[\Phi\Phi] - \frac{2}{9}(\mathrm{Tr}[\Phi])^2\right)$
	because of the Z_2	only phenomenolgical

It is our next task to investigate how we make those concepts cooperative.

1982□ Start of the problem:
Proposal of the mass formula of the charged leptons

1990: Prototype of the scalar potential model
Understanding from the VEV relation

1992: Observation of the new tau mass which was
in excellent agreement with the prediction in 1982

2006: Scalar potential model with harmless V_{SB} based on S_3

2007: Superpotential model based on U(3) flavor

Time seems to shed on this problem little by little

Thank you