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Broken SU(3) Flavor Symmetry and Tribimaximal Neutrino Mixing

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I
Introduction

Lepton sector is a treasure house of clues to unification model of quarks and leptons

The observed facts tell us:

[1] Beautiful charged lepton mass relation

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \qquad (1)$$

[2] Suggestive form of the observed neutrino mixing, so-called "Tribimaximal mixing"

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad (2)$$

These relations seem to be highly related to a permutation symmetry S_3

Permutation symmetry S_3

We define the doublet (ϕ_π, ϕ_η) and singlet ϕ_σ of S_3:

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \quad (3)$$

from the three objects (ϕ_1, ϕ_2, ϕ_3)

[1] How about Charged Leptons

(i) We assume an S_3 invariant potential

$$V(\phi) = m^2(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)$$

$$+\lambda_1(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)^2 + \lambda_2\phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2) \quad (5)$$

Then the minimizing condition of V leads to

$$v_\pi^2 + v_\eta^2 = v_\sigma^2 \quad (6)$$

which means the VEV relation

$$v_1^2+v_2^2+v_3^2 = v_\pi^2+v_\eta^2+v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1+v_2+v_3}{\sqrt{3}}\right)^2 . \quad (7)$$

(ii) Therefore, if we assume a model

$$H_e^{eff} = \sum_i \bar{e}_{Li} (\phi_i)^2 e_{Ri} \tag{8}$$

we can obtain
the charged lepton mass relation (1)

However, note that the potential (5) and the effective Hamiltonian (8) are S_3 invariant, but those are not S_3 invariant general forms.
We must consider more strong constraints.
We will assume a broken SU(3) symmetry.

[2] How about Neutrino Mixing

From the definition (3), we obtain

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \psi_\eta \\ \psi_\sigma \\ \psi_\pi \end{pmatrix} . \quad (10)$$

This is just the tribimaximal mixing.

We must consider that

(ψ_1, ψ_2, ψ_3) : the weak eigenstates

$(\psi_\eta, \psi_\sigma, \psi_\pi)$: the mass eigenstates

- However, we need the mass spectrum

$$m_\eta \simeq m_\sigma \ll m_\pi$$

(or $m_\pi \ll m_\eta \simeq m_\sigma$)

- How we can obtain such a neutrino mass spectrum?

- We will propose a model in which the right-handed neutrinos are SU(3) flavor singlets.

II
How to Get the VEV Relation

The idea that the mass spectrum originates not in the structure of the Yukawa c.c. Y_{ij} but in the VEV structure $\langle \phi_{ij} \rangle$ of a scalar ϕ is attractive.

Since we consider that quarks and leptons are triplets under a symmetry, it is natural to consider that the scalar ϕ is a nonet (3x3) of the symmetry.

We consider S_4 which is embedded into SU(3)

	SU(3)	S4	
ψ	1	1	ϕ_σ
ψ_i	3	3'	$\ell_{Li} = (\nu_i, e_i)_L,\ e_{Ri}$
$\psi_{(ij)}$	6	1+2+3	
ψ_i^j	8	2+3+3'	(ϕ_π, ϕ_η)
$\psi_{(ijk)}$	10	1'+3+3'+3'	

We assume that **1+2 of S4 originate in 1+8 of SU(3)**

Basic assumption

The fields ϕ_d and ϕ_u always appears in the theory in terms of the nonet form

$$\phi = \begin{pmatrix} \phi_1^1 & * & * \\ * & \phi_2^2 & * \\ * & * & \phi_3^3 \end{pmatrix} \tag{9}$$

$$\begin{aligned} \phi_1^1 &= \frac{1}{\sqrt{3}}\phi_\sigma + \frac{2}{\sqrt{6}}\phi_\eta \\ \phi_2^2 &= \frac{1}{\sqrt{3}}\phi_\sigma - \frac{1}{\sqrt{6}}\phi_\eta - \frac{1}{\sqrt{2}}\phi_\pi \\ \phi_3^3 &= \frac{1}{\sqrt{3}}\phi_\sigma - \frac{1}{\sqrt{6}}\phi_\eta + \frac{1}{\sqrt{2}}\phi_\pi \end{aligned} \tag{10}$$

We assume the following U(3) invariant superpotential

$$W = m\mathrm{Tr}(\phi\phi) + \lambda\mathrm{Tr}(\phi\phi\phi) \quad (11)$$

However, the condition $\partial W/\partial\phi = 0$ cannot lead to a desirable VEV relation.

We assume a Z_2 parity for

$$\phi = \phi^{(8)} + \phi^{(1)}$$

- + (12)

Tr[(9)(9)] = Tr[(8)(8) + (1)(1)] : no effect of Z_2

-   -   +   +

Tr[(9)(9)(9)]=Tr[(8)(8)(8) + 3(1)(8)(8) +(1)(1)(1)]

- - -   + - -   + + +

forbidden

By using $\phi^{(1)} = \frac{1}{3}\mathrm{Tr}\phi$, we can rewrite

$$\mathrm{Tr}(\phi\phi\phi) \Rightarrow \mathrm{Tr}(3\phi^{(1)}\phi^{(8)}\phi(8) + \phi^{(1)}\phi^{(1)}\phi^{(1)})$$

$$= \mathrm{Tr}(\phi)\left[\mathrm{Tr}(\phi\phi) - \frac{2}{9}(\mathrm{Tr}\phi)^2\right] \quad (13)$$

From the modified W under Z_2, we obtain

$$\frac{\partial W}{\partial \phi} = 0 = f_1 \phi + f_0 \mathbf{1} \qquad (14)$$

$$f_1 = 2(m + \lambda \mathrm{Tr}\phi) \qquad (15)$$

$$f_0 = \lambda \left[\mathrm{Tr}(\phi\phi) - \frac{2}{3}(\mathrm{Tr}\phi)^2 \right] \qquad (16)$$

In order that there exists a solution $\phi \neq 0$, $f_1=0$ and $f_0=0$ are required. The requirement $f_0=0$ just means

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2 \qquad (17)$$

for the diagonal basis of $\langle \phi \rangle = \mathrm{diag}(v_1, v_2, v_3)$

III
How to Get Masses and Mixing

We assume a Frogatt-Nielsen type effective Hamiltonian

$$H^{eff} = y_e \bar{\ell}_L H_L^d \frac{\phi_d}{\Lambda} \frac{\phi_d}{\Lambda} \frac{\xi}{\Lambda} e_R + y_\nu \bar{\ell}_L H_L^u \frac{\phi_u}{\Lambda} \frac{\chi}{\Lambda} \nu_R + y_R \overline{\nu}_R \Phi_R \nu_R^* \quad (18)$$

(For a model without Froggatt-Nielsen,
see Haba and Koide in preparation.)

SU(3) and S4 assignments

Fields	$SU(2)_L$	SU(3)	S_4	Z_3	Z_3'	Z_2
ℓ_L	**2**	**3**	$\mathbf{3'}$	0	0	0
e_R	**1**	**3**	$\mathbf{3'}$	0	0	0
$\nu_R^{(\pm)}$	**1**	**1**	$\mathbf{1}$	0	0	0/+1
ϕ_u	**1**	**1+8**	$1+(\mathbf{2}+\mathbf{3}+\mathbf{3'})$	+1	+1	0/+1
ϕ_d	**1**	**1+8**	$1+(\mathbf{2}+\mathbf{3}+\mathbf{3'})$	−1	−1	0/+1
$\xi^{(\pm)}$	**1**	**1**	1	0	−1	0/+1
χ	**1**	**3**	$\mathbf{3'}$	+1	−1	0
H_L^u	**2**	**1**	$\mathbf{1}$	+1	0	0
H_L^d	**2**	**1**	$\mathbf{1}$	−1	0	0
Φ_R	**1**	**1**	$\mathbf{1}$	0	0	0

Charged lepton sector

We introduce a new scalar $\xi = \frac{1}{\sqrt{2}}(\xi^{(+)} + \xi^{(-)})$

in order to recover the dropped term (1)(8)

in (9)(9)=[(8)(8)+(1)(1)+2 (1)(8)]:

$$H_e^{eff} = \frac{y_e}{\sqrt{2}} \bar{e}_L [(\phi_d^{(8)} \phi_d^{(8)} + \phi_d^{(1)} \phi_d^{(1)}) \xi^{(+)}$$

$$+(\phi_d^{(8)} \phi_d^{(1)} + \phi_d^{(1)} \phi_d^{(8)}) \xi^{(-)}] e_R$$

$$= \frac{y_e v_d v_\xi}{\sqrt{2}\Lambda^3} \sum_i \bar{e}_L^i \langle (\phi_d)_{ii} \rangle^2 e_{Ri}, \tag{19}$$

so that we have obtained the mass relation (1),

where we have assumed $\langle \xi^{(+)} \rangle = \langle \xi^{(-)} \rangle \equiv v_\xi$ (20)

Neutrino sector

Note that the right-handed neutrinos in the present model are flavor singlets :

$$H_{Dirac}^{eff} = y_\nu \frac{v_u}{\Lambda^2} \bar{\nu}_L^i \langle (\phi_u)_i^j \rangle \langle \chi_j \rangle (\nu_R^{(+)} + \nu_R^{(-)})$$

$$= y_\nu \frac{v_u v_\chi}{\sqrt{2}\Lambda^2} (\bar{\nu}_\eta \; \bar{\nu}_\sigma \; \bar{\nu}_\pi)_L \left[\begin{pmatrix} v_\eta \\ 0 \\ v_\pi \end{pmatrix} \nu_R^{(-)} + \begin{pmatrix} 0 \\ v_\sigma \\ 0 \end{pmatrix} \nu_R^{(+)} \right] \quad (21)$$

where we have assumed $\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle \equiv v_\chi$

Then we obtain

$$U_{TB}^T M_\nu U_{TB} \equiv M_\nu^{(\eta\sigma\pi)} = \frac{1}{M_R^{(-)}} \begin{pmatrix} v_\eta^2 & 0 & v_\pi v_\eta \\ 0 & 0 & 0 \\ v_\pi v_\eta & 0 & v_\pi^2 \end{pmatrix} + \frac{1}{M_R^{(+)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_\sigma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (22)$$

For $v_\pi = 0$, we obtain the tribimaximal neutrino mixing matrix U_{TB} and the mass spectrum

$$m_{\nu 1} = k v_\eta^2, \quad m_{\nu 2} = k v_\sigma^2 \qquad (23)$$

in the limit of $M_R^{(+)} = M_R^{(-)} \equiv M_R$

Note that the result (23) gives an inverse mass hierarchy, so that the effective electron neutrino mass is given by

$$\langle m_{\nu_e} \rangle = \left| \sum_i U_{ei}^2 m_{\nu i} \right| \simeq |m_{\nu 1}| \simeq |m_{\nu 2}| \simeq \sqrt{\Delta m_{atm}^2} = 5.23^{+0.25}_{-0.40} \times 10^{-2}\,\mathrm{eV} \qquad (24)$$

IV Concluding Remarks

- By requiring the Z_2 invariance, we have obtained a beautiful VEV relation (6) [(7)].
- However, because of the requirement of Z_2, the model which gives $m_{ei} \propto v_i^2$ becomes somewhat complicated. This will be improved in a future version.
- It is worth while noting the idea that the lepton doublets and right-handed charged leptons, while the right-handed neutrinos are flavor singlets.