

# $S_4$ Flavor Symmetry Embedded into SU(3) and Lepton Masses and Mixing

Yoshio Koide

*Institute for Higher Education Research and Practice, Osaka University,*

*1-16 Machikaneyama, Toyonaka, Osaka 560-0043, Japan*

*E-mail address: koide@het.phys.sci.osaka-u.ac.jp*

## Abstract

Under an assumption that an  $S_4$  flavor symmetry is embedded into SU(3), a lepton mass matrix model is investigated. A Froggatt-Nielsen type model is assumed, and the flavor structures of the masses and mixing are caused by VEVs of SU(2)<sub>L</sub>-singlet scalars  $\phi_u$  and  $\phi_d$  which are nonets ( $\mathbf{8}+\mathbf{1}$ ) of the SU(3) flavor symmetry, and which are broken into  $\mathbf{2}+\mathbf{3}+\mathbf{3}'$  and  $\mathbf{1}$  of  $S_4$ . If we require the invariance under the transformation  $(\phi^{(8)}, \phi^{(1)}) \rightarrow (-\phi^{(8)}, +\phi^{(1)})$  for the superpotential of the nonet field  $\phi^{(8+1)}$ , the model leads to a beautiful relation for the charged lepton masses. The observed tribimaximal neutrino mixing is understood by assuming two SU(3) singlet right-handed neutrinos  $\nu_R^{(\pm)}$  and an SU(3) triplet scalar  $\chi$ .

## 1 Introduction

The observed mass spectra and mixings of the fundamental particles will provide promising clues to unified understanding of the quarks and leptons. Especially, in the lepton sector, the following characteristic features have been observed [1]:

- (i) The observed charged lepton masses  $(m_e, m_\mu, m_\tau)$  satisfy the relation [2, 3]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \quad (1.1)$$

with remarkable precision;

- (ii) The observed neutrino mixing  $U_\nu$  is nearly given by the so-called tribimaximal mixing [4]

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1.2)$$

Such characteristic features have not been seen in the quark sector. For example, the mixing form (1.2) suggests that the mixing can be described by Clebsh-Gordan-like coefficients, while, for the Cabibbo-Kobayashi-Maskawa mixing in the quark sector, such a characteristic feature has not been seen, although we have known some relations among the mixing angles and quark mass ratios. Therefore, for a start, in the present paper, we investigate the lepton masses and mixings.

In order to understand the relation (1.1), for example, we assume that there are three scalars  $\phi_i$  ( $i = 1, 2, 3$ ), and the values of the charged lepton masses  $m_{ei}$  are proportional to the square of the vacuum expectation values (VEVs)  $v_i = \langle \phi_i \rangle$  of the scalars  $\phi_i$ ,  $m_{ei} = kv_i^2$  (in the

Ref.[3, 5, 6], for instance, a seesaw type model  $(M_e)_{ij} = \delta_{ij}v_i(M_E)^{-1}v_j$  has been assumed). We define singlet  $\phi_\sigma$  and doublet  $(\phi_\pi, \phi_\eta)$  of a permutation symmetry  $S_3$  [7] by

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \quad (1.3)$$

from the three objects  $(\phi_1, \phi_2, \phi_3)$ , and we consider the following  $S_3$  invariant scalar potential  $V(\phi)$  [3, 8, 9]:

$$V(\phi) = m^2(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2) + \lambda_1(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)^2 + \lambda_2\phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2). \quad (1.4)$$

The minimizing condition of the potential (1.4) leads to the relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2. \quad (1.5)$$

The relation (1.5) means

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2, \quad (1.6)$$

because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2. \quad (1.7)$$

Therefore, we can obtain the mass relation (1.1). Here, note that although the scalar potential (1.4) is invariant under the  $S_3$  symmetry, but it is not a general one of the  $S_3$  invariant form. As pointed out in Ref. [9], the scalar potential with a general form cannot lead to the relation (1.5). For the derivation of the VEV relation (1.5), it is essential to choose the specific form (1.4) of the  $S_3$  invariant terms. Similar formulation is also possible for other discrete symmetries  $A_4$  [10] and  $S_4$  (see below). However, in such a symmetry, we still need an additional specific selection rule. What is the meaning of such a specific selection? In the present paper, we investigate this problem by assuming that the  $S_4$  flavor symmetry is embedded into  $SU(3)$ .

Recently, a superpotential which leads to the relation (1.5) has proposed by Ma [11] on the basis of a symmetry  $\Sigma(81)$ . Stimulated by the Ma's idea, the author [10] has also investigated a similar superpotential on the basis of a symmetry  $A_4$ . Here, based on an  $S_4$  flavor symmetry instead of the  $A_4$  symmetry, let us review the superpotential  $W$  which gives the relation (1.5). We denote singlet and doublet of  $S_4$  as  $\phi_\sigma$  and  $\phi_D = (\phi_\pi, \phi_\eta)^T$ , respectively, as well as those in  $S_3$ . In order to write the superpotential for the scalar fields  $\phi_\sigma$  and doublet  $\phi_D$  of  $S_4$ , we put the following phenomenological rule [10]: the field  $\phi_a$  ( $a = \sigma, D$ ) to the power  $n$ th,  $(\phi_a)^n$  ( $n = 1, 2, 3$ ), appears always accompanied with the factor  $1/n!$  in the superpotential  $W$ . Under this phenomenological rule, we can uniquely write the superpotential of  $\phi_\sigma$  and  $\phi_D$  as

$$W(\phi) = \frac{1}{2!}m(\phi_\sigma^2 + \phi_D^T\phi_D) + \lambda\left(\frac{1}{2!}\phi_\sigma\phi_D^T\phi_D + \frac{1}{3!}\phi_\sigma^3\right)$$

$$= \frac{1}{2}m(\phi_\sigma^2 + \phi_\pi^2 + \phi_\eta^2) + \frac{1}{2}\lambda \left[ (\phi_\pi^2 + \phi_\eta^2)\phi_\sigma + \frac{1}{3}\phi_\sigma^3 \right]. \quad (1.8)$$

The potential (1.8) can also lead the relation (1.5). What is the meaning of this phenomenological rule?

On the other hand, if we assume that the charged lepton fields  $e_{Li}$  and  $e_{Ri}$  are triplets of  $S_4$ , and we assume a Froggatt-Nielsen type model [12],  $H_e^{eff} = (\bar{\ell}_L H_L^d (\phi_\sigma, \phi_D)^2 e_R)$ , where  $\ell_{Li}$  are the  $SU(2)_L$  doublet left-handed lepton fields  $\ell_{Li} = (\nu_i, e_i)_L$  and  $H_d$  is the conventional  $SU(2)_L$  doublet Higgs scalar  $H_L^d = (H_L^-, H_L^0)$ , then we obtain

$$\begin{aligned} H_e^{eff} &= (\bar{e}_L e_R)_\sigma [\phi_\sigma \phi_\sigma + \sqrt{2}(\phi_D \phi_D)_\sigma] \\ &+ (\bar{e}_L e_R)_\pi [2\phi_\sigma \phi_\pi - (\phi_D \phi_D)_\pi] + (\bar{e}_L e_R)_\eta [2\phi_\sigma \phi_\eta - (\phi_D \phi_D)_\eta] \\ &= \sqrt{3} [\bar{e}_{L1}(\phi_1)^2 e_{R1} + \bar{e}_{L2}(\phi_2)^2 e_{R3} + \bar{e}_{L3}(\phi_3)^2 e_{R3}], \end{aligned} \quad (1.9)$$

where we have dropped the  $SU(2)_L$  structure. Here,  $(\bar{e}_L e_R)_\sigma$  and  $((\bar{e}_L e_R)_\pi, (\bar{e}_L e_R)_\eta)$  are singlet and doublet composed of the triplets  $(e_{L1}, e_{L2}, e_{L3})$  and  $(e_{R1}, e_{R2}, e_{R3})$ , respectively, and those are given by

$$\begin{aligned} (\bar{e}_L e_R)_\sigma &= \frac{1}{\sqrt{3}} (\bar{e}_{L1} e_{R1} + \bar{e}_{L2} e_{R2} + \bar{e}_{L3} e_{R3}), \\ (\bar{e}_L e_R)_\pi &= \frac{1}{\sqrt{2}} (-\bar{e}_{L2} e_{R2} + \bar{e}_{L3} e_{R3}), \\ (\bar{e}_L e_R)_\eta &= \frac{1}{\sqrt{6}} (2\bar{e}_{L1} e_{R1} - \bar{e}_{L2} e_{R2} - \bar{e}_{L3} e_{R3}). \end{aligned} \quad (1.10)$$

Also,  $(\phi_D \phi_D)_\sigma$  and  $((\phi_D \phi_D)_\pi, (\phi_D \phi_D)_\eta)$  are singlet and doublet composed of the doublet  $\phi_D = (\phi_\pi, \phi_\eta)$ , respectively, and those are defined by

$$\begin{aligned} (\phi_D \phi_D)_\sigma &= \frac{1}{\sqrt{2}} (\phi_\pi \phi_\pi + \phi_\eta \phi_\eta), \\ (\phi_D \phi_D)_\pi &= \frac{1}{\sqrt{2}} (\phi_\pi \phi_\eta + \phi_\eta \phi_\pi), \\ (\phi_D \phi_D)_\eta &= \frac{1}{\sqrt{2}} (\phi_\pi \phi_\pi - \phi_\eta \phi_\eta), \end{aligned} \quad (1.11)$$

Thus, we can obtain the relation (1.1) for the charged lepton masses from Eqs.(1.6) and (1.9). However, in Eq.(1.9), we have again assumed some specific combinations of the  $S_4$  invariant terms. What is the meaning of the selection rule?

Now, let us return the topic of the tribimaximal mixing. From the definition (1.2), we can denote the fields  $(\psi_1, \psi_2, \psi_3)$  as

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = U_{TB} \begin{pmatrix} \psi_\eta \\ \psi_\sigma \\ \psi_\pi \end{pmatrix}. \quad (1.12)$$

The observed neutrino mixing (1.2) means that when the mass eigenstates of the charged leptons are given by the  $(\psi_1, \psi_2, \psi_3)$  basis, the mass eigenstates of the neutrinos are given by the

$(\psi_\eta, \psi_\sigma, \psi_\pi)$  basis. Therefore, the problem is to find a model where the charged lepton mass eigenstates are  $(e_1, e_2, e_3)$ , while the neutrino mass eigenstates are given by  $(\nu_\eta, \nu_\sigma, \nu_\pi)$  with the mass hierarchy  $m_\eta^2 < m_\sigma^2 \ll m_\pi^2$  (or  $m_\pi^2 \ll m_\eta^2 < m_\sigma^2$ ). In the present paper, we will investigate such a model based on an  $S_4$  model.

Note that the fermions  $(\psi_1, \psi_2, \psi_3)$  is a triplet of  $S_4$ , but the basis  $(\psi_\eta, \psi_\sigma, \psi_\pi)$  is not in any irreducible representations of  $S_4$ , while the scalar  $(\phi_1, \phi_2, \phi_3)$  is not irreducible representation of  $S_4$ , but  $(\phi_\pi, \phi_\eta)$  and  $\phi_\sigma$  are doublet and singlet of  $S_4$ .

Thus, the characteristic features (1.1) and (1.2) in the lepton sector may be understood from the language of  $S_4$  (also  $S_3$  or  $A_4$ ). However, as seen from the above review, the characteristic features (1.1) and (1.2) cannot be understood from the  $S_4$  symmetry only. We need some additional assumptions. In this paper, we will investigate these problems under an assumption that the present  $S_4$  symmetry is embedded into an  $SU(3)$  symmetry.

## 2 VEVs of $SU(3)$ nonet scalars

The goal in the present section is to obtain the VEV relation (1.6) [i.e. (1.5)]. As seen in the previous section, in order to obtain the desirable results (1.5), we need assume an equal weight between the doublet and singlet terms of  $S_4$ . In the present paper, we assume that the  $S_4$  symmetry is embedded into an  $SU(3)$  symmetry, and we consider that the doublet  $(\phi_\pi, \phi_\eta)$  and singlet  $\phi_\sigma$  of  $S_4$  come from  $SU(3)$  octet and singlet, respectively [13]. The essential assumption in the present paper is that the fields  $\phi_u$  and  $\phi_d$  always appear in the theory with the form of the nonet of  $U(3)$ :

$$\phi = \begin{pmatrix} \phi_1^1 & * & * \\ * & \phi_2^2 & * \\ * & * & \phi_3^3 \end{pmatrix}, \quad (2.1)$$

where

$$\begin{aligned} \phi_1^1 &= \frac{1}{\sqrt{3}}\phi_\sigma + \frac{2}{\sqrt{6}}\phi_\eta, \\ \phi_2^2 &= \frac{1}{\sqrt{3}}\phi_\sigma - \frac{1}{\sqrt{6}}\phi_\eta - \frac{1}{\sqrt{2}}\phi_\pi, \\ \phi_3^3 &= \frac{1}{\sqrt{3}}\phi_\sigma - \frac{1}{\sqrt{6}}\phi_\eta + \frac{1}{\sqrt{2}}\phi_\pi, \end{aligned} \quad (2.2)$$

and the index  $f$  ( $f = u, d$ ) has been dropped.

The outline to obtain the superpotential form (1.8) in the present scenario is as follows: The  $SU(3)$  invariant superpotential for the nonet fields  $\phi_f$  ( $f = u, d$ ) are given by

$$W(\phi) = \frac{1}{2}m\text{Tr}(\phi\phi) + \frac{1}{2\sqrt{3}}\lambda\text{Tr}(\phi\phi\phi), \quad (2.3)$$

where, for convenience, we have dropped the index  $f$ . Although the term  $\text{Tr}(\phi\phi)$  gives the desirable term  $\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2 + \dots$  of  $S_4$ , the cubic term  $\text{Tr}(\phi\phi\phi)$  gives

$$\text{Tr}(\phi\phi\phi) = \sqrt{3} \left[ \frac{1}{\sqrt{2}} \left( -\phi_\pi^2 + \frac{1}{3}\phi_\eta^2 \right) \phi_\eta + (\phi_\pi^2 + \phi_\eta^2)\phi_\sigma + \frac{1}{3}\phi_\sigma^3 \right] + \dots, \quad (2.4)$$

where the terms “...” denote terms which include  $\mathbf{3}$  and  $\mathbf{3}'$  of the subgroup  $S_4$ , so that the potential (2.3) cannot give the relation (1.5). Therefore, we must drop the first term in cubic terms (2.4). For this purpose, we introduce  $Z_2$  symmetry, and we assign the  $Z_2$  parity  $-1$  and  $+1$  for the octet part  $\phi^{(8)}$  and singlet part  $\phi^{(1)}$  of the nonet field  $\phi$ , respectively. The symmetry  $Z_2$  breaks  $U(3)$  into  $SU(3)$ . Under the requirement of the  $Z_2$  invariance, i.e. the invariance under the transformation

$$(\phi^{(8)}, \phi^{(1)}) \rightarrow (-\phi^{(8)}, +\phi^{(1)}), \quad (2.5)$$

the terms  $\text{Tr}(\phi^{(8)}\phi^{(8)}\phi^{(8)})$ , i.e.  $(-\phi_\pi^2 + (1/3)\phi_\eta^2)\phi_\eta + \dots$ , are forbidden. The revised superpotential is not  $U(3)$  invariant anymore, but it is still  $SU(3)$  invariant. In other words, in the present model, the flavor symmetry  $U(3)$  is broken from the beginning by the  $Z_2$  symmetry.

The superpotential (2.3) with the constraint (2.5) gives the form (1.8) which leads to the VEV relation (1.5). However, we want to assign  $Z_3$  charges to the fields  $\phi_f$  in order to select some special combinations among the fields in the present model as we see in the next section. If we assign  $Q(Z_2) = +1$  to the field  $\phi$ , the term  $\lambda\phi\phi\phi$  is allowed, but the term  $m\phi\phi$  is forbidden by the  $Z_3$  symmetry. We need the  $m$ -term, but also need the  $Z_3$  assignment. Therefore, in the present paper, keeping the idea of (2.3) with (2.5), we propose the following super potential

$$W(\phi) = m\text{Tr}(\phi_u\phi_d) + \lambda_u\text{Tr}(\phi_u\phi_u\phi_u) + \lambda_d\text{Tr}(\phi_d\phi_d\phi_d), \quad (2.6)$$

where the fields  $\phi_u$  and  $\phi_d$  have the charges of the  $Z_3$  transformation,  $+1$  and  $-1$ , respectively.

Under the constraint (2.5), the superpotential (2.6) becomes

$$\begin{aligned} W(\phi) = m \left[ \text{Tr}(\phi_u^{(8)}\phi_d^{(8)}) + \phi_{u\sigma}\phi_{d\sigma} \right] + \sqrt{3}\lambda_u\phi_{u\sigma} \left[ \text{Tr}(\phi_u^{(8)}\phi_u^{(8)}) + \frac{1}{3}\phi_{u\sigma}^2 \right] \\ + \sqrt{3}\lambda_d\phi_{d\sigma} \left[ \text{Tr}(\phi_d^{(8)}\phi_d^{(8)}) + \frac{1}{3}\phi_{d\sigma}^2 \right]. \end{aligned} \quad (2.7)$$

From the conditions

$$\frac{\partial W}{\partial \phi_{u\sigma}} = m\phi_{d\sigma} + \sqrt{3}\lambda_u \left[ \text{Tr}(\phi_u^{(8)}\phi_u^{(8)}) + \phi_{u\sigma}^2 \right], \quad (2.8)$$

$$\frac{\partial W}{\partial \phi_{d\sigma}} = m\phi_{u\sigma} + \sqrt{3}\lambda_d \left[ \text{Tr}(\phi_d^{(8)}\phi_d^{(8)}) + \phi_{d\sigma}^2 \right], \quad (2.9)$$

$$\frac{\partial W}{\partial (\phi_u^{(8)})_i^j} = m(\phi_d^{(8)})_j^i + 2\sqrt{3}\lambda_u\phi_{u\sigma}(\phi_u^{(8)})_j^i, \quad (2.10)$$

$$\frac{\partial W}{\partial (\phi_d^{(8)})_i^j} = m(\phi_u^{(8)})_j^i + 2\sqrt{3}\lambda_d\phi_{d\sigma}(\phi_d^{(8)})_j^i, \quad (2.11)$$

by eliminating  $\phi_u$  and  $m$ , we obtain

$$4\phi_{d\sigma}^4 = \left[ \text{Tr}(\phi_d^{(8)}\phi_d^{(8)}) + \phi_{d\sigma}^2 \right]^2, \quad (2.12)$$

so that we obtain the relation

$$\phi_{d\sigma}^2 = \text{Tr}(\phi_d^{(8)}\phi_d^{(8)}) = \phi_{d\pi}^2 + \phi_{d\eta}^2 + \dots, \quad (2.13)$$

when, we take the positive sign in  $2\phi_{d\sigma}^2 = \pm[\dots]$ , where “ $\dots$ ” denotes the contributions of  $\mathbf{3}$  and  $\mathbf{3}'$  of  $S_4$ .

The result (2.13) is not our goal, because the relation contains the VEVs of the  $\mathbf{3}$  and  $\mathbf{3}'$  of  $S_4$ . So far, we have not discussed the splitting among the  $S_4$  multiplets. Now, we bring a symmetry breaking of  $SU(3)$  into  $S_4$  with an infinitesimal parameter  $\varepsilon$

$$\text{Tr}(\phi_u^{(8)}\phi_d^{(8)}) \Rightarrow \phi_{u\pi}\phi_{d\pi} + \phi_{u\eta}\phi_{d\eta} + (1 + \varepsilon) \sum_{i \neq j} (\phi_u^{(8)})_i^j (\phi_d^{(8)})_j^i, \quad (2.14)$$

by hand. (At present, we do not consider the origin of the symmetry breaking. We always consider only the limit of  $\varepsilon \rightarrow 0$ .) Then, we can obtain the solution

$$\langle (\phi_d^{(8)})_j^i \rangle = 0 \quad \text{for all } i \neq j. \quad (2.15)$$

Therefore, we can obtain the desirable relation (1.5). (However, it is possible that we can also obtain another solution with  $\phi_\pi = \phi_\eta = 0$  and  $(\phi_d^{(8)})_j^i \neq 0$ . The VEV solutions are not unique. The result (1.5) is merely one of the possible solutions.)

### 3 Model

If we regard the scalars  $\phi_i$  as  $SU(2)_L$  doublets, such a model with multi-Higgs doublets causes a flavor changing neutral current (FCNC) problem. Therefore, in the present paper, we assume a Froggatt-Nielsen [12] type model

$$H^{eff} = y_e \bar{\ell}_L H_L^d \frac{\phi_d}{\Lambda} \frac{\phi_d}{\Lambda} \frac{\xi}{\Lambda} e_R + y_\nu \bar{\ell}_L H_L^u \frac{\phi_u}{\Lambda} \frac{\chi}{\Lambda} \nu_R + y_R \bar{\nu}_R \Phi_R \nu_R^*, \quad (2.1)$$

where  $\ell_{iL}$  are  $SU(2)_L$  doublet leptons  $\ell_{iL} = (\nu_{iL}, e_{iL})$ ,  $H_L^d$  and  $H_L^u$  are conventional  $SU(2)_L$  doublet Higgs scalars,  $\phi_f$  ( $f = u, d$ ),  $\xi$  and  $\chi$  are  $SU(2)_L$  singlet scalars, and  $\Lambda$  is a scale of the effective theory. We consider that  $\langle \phi_f \rangle / \Lambda$ ,  $\langle \xi \rangle / \Lambda$  and  $\langle \chi \rangle / \Lambda$  are of the order of 1. The scalar  $\Phi_R$  has been introduced in order to generate the Majorana mass  $M_R$  of the right-handed neutrino  $\nu_R$ . The role of  $\xi$  and  $\chi$  will be explained later. In order to understand the appearance of the combinations  $H_L^d \phi_d \phi_d \xi$  and  $H_L^u \phi_u \chi$ , we assume that the scalars  $H_L^d$  and  $\phi_d$  have charge +1 of a  $Z_3$  symmetry (we denote it by  $Z_3^d$  in Table 1), and the scalars  $H_L^u$ ,  $\phi_u$  and  $\chi$  have charge +1 of another  $Z_3$  (we denote it by  $Z_3^u$  in Table 1). In order to forbid the combinations  $H_L^d \phi_d \phi_d \xi^n$  ( $n \neq 1$ ), we also assume another  $Z_3$  symmetry ( $Z_3^{fd}$ ).

On the other hand, in the charged lepton sector, we consider a Froggatt-Nielsen type interaction  $H_e^{eff} = \bar{\ell}_L H_L^d \phi_d \phi_d e_R$ . Recall that we have already assumed the invariance of the superpotential under the transformation (3.3) in order to drop the cubic part of the octet  $\phi^{(8)}$ . Therefore, the term  $\phi\phi$  means  $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)}$  under the  $Z_2$  invariance. However, in order to

**Table 1** SU(3) and S<sub>4</sub> assignments of the fields

Fields	SU(3)	S <sub>4</sub>	Z <sub>3</sub> <sup>d</sup>	Z <sub>3</sub> <sup>d'</sup>	Z <sub>3</sub> <sup>u</sup>	Z <sub>2</sub>
$\ell_L$	<b>3</b>	<b>3'</b>	0	0	0	0
$e_R$	<b>3</b>	<b>3'</b>	0	0	0	0
$\nu_R^{(\pm)}$	<b>1</b>	<b>1</b>	0	0	0	0/+1
$\phi_d$	<b>1+8</b>	<b>1 + (2 + 3 + 3')</b>	+1	+1	0	0
$\xi^{(\pm)}$	<b>1</b>	<b>1</b>	0	+1	0	0/+1
$\phi_u$	<b>1+8</b>	<b>1 + (2 + 3 + 3')</b>	0	0	+1	0
$\chi$	<b>3</b>	<b>3'</b>	0	0	+1	0
$\Phi_R$	<b>1</b>	<b>1</b>	0	0	0	0
$H_L^d$	<b>1</b>	<b>1</b>	+1	0	0	0
$H_L^u$	<b>1</b>	<b>1</b>	0	0	+1	0

give  $m_{ei} \propto \langle \phi_i^i \rangle^2$ , what we want is not  $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)}$ , but  $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)} + \phi^{(8)}\phi^{(1)} + \phi^{(1)}\phi^{(8)}$ . In order to evade this problem, we have introduced additional fields  $\xi^{(+)}$  and  $\xi^{(-)}$  whose Z<sub>2</sub> parity are +1 and -1, respectively. The revised interactions are given by

$$H_e^{eff} = y_e \bar{e}_L^i (\phi_d)_i^j (\phi_d)_j^k (\xi^{(+)} + \xi^{(-)}) e_{Rk}, \quad (3.4)$$

where we have dropped the Higgs scalar  $H_L^d$  since we discuss flavor structure only. The expression (3.4) becomes

$$H_e^{eff} = y_e \bar{e}_L [(\phi_d^{(8)} \phi_d^{(8)} + \phi_d^{(1)} \phi_d^{(1)}) \xi^{(+)} + (\phi_d^{(8)} \phi_d^{(1)} + \phi_d^{(1)} \phi_d^{(8)}) \xi^{(-)}] e_R. \quad (3.5)$$

Since we have assumed that  $\xi^{(+)}$  and  $\xi^{(-)}$  appear symmetrically in the theory, we also assume

$$\langle \xi^{(+)} \rangle = \langle \xi^{(-)} \rangle \equiv v_\xi. \quad (3.6)$$

Then, we obtain the effective Hamiltonian for the charged leptons

$$H_e^{eff} = \frac{y_e v_d v_\xi}{\Lambda^3} \sum_i \bar{e}_L^i \langle (\phi_d^{(8+1)})_i \rangle^2 e_{Ri}, \quad (3.7)$$

where  $v_d = \langle H_L^{d0} \rangle$ . Since the fields  $(\phi_d)_i^i$  are given by Eq.(2.3), we obtain the charged lepton mass relation (1.1) from the VEV relation (1.6).

However, the present mechanism to obtain  $m_{ei} \propto \langle \phi_i^i \rangle^2$  is somewhat artificial. The present mechanism will be improved in the future model. [Of course, there is a possibility that the superpotential (3.1) must exactly be invariance under the Z<sub>2</sub> symmetry, but the effective Hamiltonian (3.3) does not need to be invariance under the Z<sub>2</sub> symmetry. Then, we can consider a model without  $\xi^{(\pm)}$ .]

By the way, from the superpotential (1.8), we can obtain the VEV relation (1.5) only. In order to fix the three charged lepton masses completely, we must fix the ratio  $v_\pi/v_\eta$ . The value  $v_\pi/v_\eta$  will be fixed by introducing an SU(3) (or S<sub>4</sub>) symmetry breaking in the mass term of  $W$ . In Sec.5, we will demonstrate that we can indeed choose such a soft symmetry breaking term which does not spoil the VEV relation (1.5).

## 4 Neutrino mixing

In the present model, the right-handed neutrinos  $\nu^{(\pm)}$  are singlets of SU(3). Therefore, the structure of the neutrino mass matrix  $M_\nu = m_L^\nu M_R^{-1} (m_L^\nu)^T$  is essentially determined by the structure of the neutrino Dirac mass matrix  $m_L^\nu$ . In order to compensate for the absence of the conventional triplet neutrinos  $\nu_R$ , a new scalar  $\chi$  which is a triplet of SU(3) has been introduced. The neutrino Dirac mass terms are given by the following effective Hamiltonian

$$H_{Dirac}^{eff} = y_\nu \frac{v_u}{\Lambda^2} \bar{\nu}_L^i \langle (\phi_u)^j \rangle \langle \chi_j \rangle (\nu_R^{(+)} + \nu_R^{(-)}), \quad (4.1)$$

where  $v_u = \langle H_L^{u0} \rangle$ . It is likely that the scalar potential  $V(\chi)$  for the SU(3) triplet  $\chi$  has a specific VEV solution

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle \equiv v_\chi. \quad (4.2)$$

When we assume the VEVs (4.2), we obtain

$$H_{Dirac}^{eff} = y_\nu \frac{v_u v_\chi}{\Lambda^2} (\bar{\nu}_\eta \ \bar{\nu}_\sigma \ \bar{\nu}_\pi)_L \left[ \begin{pmatrix} v_\eta \\ 0 \\ v_\pi \end{pmatrix} \nu_R^{(-)} + \begin{pmatrix} 0 \\ v_\sigma \\ 0 \end{pmatrix} \nu_R^{(+)} \right], \quad (4.3)$$

where  $v_a = \langle \phi_{ua} \rangle$  ( $a = \pi, \eta, \sigma$ ) (for convenience, we have dropped the index  $u$ ). Therefore, we obtain the effective neutrino mass matrix on the  $(\eta, \sigma, \pi)$  basis,

$$M_\nu^{(\eta\sigma\pi)} = \frac{1}{M_R^{(-)}} \begin{pmatrix} v_\eta^2 & 0 & v_\pi v_\eta \\ 0 & 0 & 0 \\ v_\pi v_\eta & 0 & v_\pi^2 \end{pmatrix} + \frac{1}{M_R^{(+)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_\sigma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (4.4)$$

where  $M_R^{(\pm)} = y_R^{(\pm)} \langle \Phi_R \rangle$ , and we have dropped the common factors  $(y_\nu v_u v_\chi / \Lambda^2)^2$ . As we state in the next section, the ratio  $v_\pi / v_\eta$  cannot be determined from the potential (1.8), and the ratio is determined by a soft  $S_4$  symmetry breaking term  $W_{SB}$ . Of course, we can choose a solution  $v_\pi = 0$  in the superpotential  $W(\phi_u)$  by choosing a suitable parameter  $\beta$  in  $W_{SB}$ , which is defined in the next section, differently from the case of  $W(\phi_d)$ . Then, the neutrino mass matrix (4.4) becomes a diagonal form  $D_\nu = (1/M_R^{(-)}) \text{diag}(v_\eta^2, 0, 0) + (1/M_R^{(+)}) \text{diag}(0, v_\sigma^2, 0)$ . Since the mass matrix on the  $(\nu_1, \nu_2, \nu_3) = (\nu_e, \nu_\mu, \nu_\tau)$  basis is given by

$$M_\nu = U_{TB} M_\nu^{(\eta\sigma\pi)} U_{TB}^T = U_{TB} D_\nu U_{TB}^T, \quad (4.5)$$

we can obtain the tribimaximal mixing

$$U_\nu = U_{TB}, \quad (4.6)$$

and the neutrino masses

$$m_{\nu 1} = k v_\eta^2, \quad m_{\nu 2} = k v_\sigma^2, \quad m_{\nu 3} = 0, \quad (4.7)$$

for the case of  $M_R^{(+)} = M_R^{(-)} \equiv M_R$ , where  $k = (y_\nu v_u v_\chi)^2 / M_R \Lambda^4$  and  $(\nu_\eta, \nu_\sigma, \nu_\pi)$  has been renamed  $(\nu_1, \nu_2, \nu_3)$  according to the conventional naming.

However, since we have taken  $v_\pi = 0$ , the value of  $v_\eta$  satisfies  $v_\eta^2 = v_\sigma^2$  from the relation (1.5), so that the result (4.8) gives  $m_{\nu 1} = m_{\nu 2}$ . The observed value  $\Delta m_{solar}^2$  is small, but it is not zero. Therefore, we must consider a small deviation between the first and second terms in (4.5) (i.e.  $M_R^{(+)} \neq M_R^{(-)}$ ). Since the value  $M_R^{(-)} / M_R^{(+)}$  is free in the present model, we cannot predict an explicit value of the ratio  $\Delta m_{solar}^2 / \Delta m_{atm}^2$ .

Since the present model gives an inverse hierarchy of the neutrino masses, the predicted effective electron neutrino mass

$$\langle m_{\nu e} \rangle = \left| \sum_i U_{ei}^2 m_{\nu i} \right| \simeq |m_{\nu 1}| \simeq |m_{\nu 2}| \simeq \sqrt{\Delta m_{atm}^2} = 5.23_{-0.40}^{+0.25} \times 10^{-2} \text{ eV}, \quad (4.8)$$

where we have used the value [14]  $\Delta m_{atm}^2 = 2.74_{-0.26}^{+0.44} \times 10^{-3} \text{ eV}^2$ . This value (4.8) is sufficiently sensitive to the next generation experiments of the neutrinoless double beta decay.

## 5 Summary

In conclusion, on the basis of the  $S_4$  symmetry which is embedded into  $SU(3)$ , we have investigated a Froggatt-Nielsen type model (2.1). In the derivation of the VEV relation (1.5), the essential assumptions for the superpotential  $W$  are the following two: (i) the scalar fields  $\phi$  always appear in terms of the nonet form of  $U(3)$ , (2.3); (ii) the superpotential  $W$  is invariant under the  $Z_2$  transformation (3.3). Then, we have obtained the VEV relation (1.5). Since the charged lepton interactions  $\bar{\ell}_L H_L^d \phi_d \phi_d e_R$  give  $m_{ei} \propto \langle (\phi_d)_i \rangle^2$ , we have obtained the charged lepton mass relation (1.1).

For the neutrino sector, we have obtained the tribimaximal mixing (1.2) by introducing an  $SU(3)$  triplet scalar  $\chi$  and the two  $SU(3)$  singlet right-handed neutrinos  $\nu_R^{(\pm)}$  in addition to the nonet scalar  $\phi_u$ . In the present model, since the right-handed neutrinos are singlets of  $SU(3)$ , the Majorana neutrino mass matrices  $M_R^{(\pm)}$  have no flavor structure. However, the requirement that  $\nu_R^{(\pm)}$  are singlets of  $SU(3)$  is essential for the structure of the neutrino mass matrix  $M_\nu = m_L^\nu M_R^{-1} (m_L^\nu)^T$  which gives the tribimaximal mixing  $U_{TB}$ . For the neutrino mass spectrum, since the model gives  $m_{\nu 1} = m_{\nu 2}$  in the limit of  $M_R^{(+)} = M_R^{(-)}$ , we must consider a small deviation  $M_R^{(+)} \neq M_R^{(-)}$ . Since the value of  $M_R^{(-)} / M_R^{(+)}$  is a free parameter in the present model, we cannot predict the value  $\Delta m_{solar}^2 / \Delta m_{atm}^2$  at present, although the smallness of the ratio  $\Delta m_{solar}^2 / \Delta m_{atm}^2$  can be understood.

The present model seems to provide suggestive hints on seeking for a model which leads to the tribimaximal mixing (1.2) and the charged lepton mass relation (1.1), although the model has still many points which should be improved. At the same time, the model will provide a clue to the quark mass matrix model from a point of unified view of the quarks and leptons. The extension of the present model to the quark mass matrix model will be given elsewhere.

## Acknowledgements

The author would like to thank E. Takasugi, T. Fukuyama and H. Fusaoka for helpful conversations. Especially, the author is much indebted to H. Fusaoka for the phase convention

of  $S_4$ . This work is supported by the Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, Japan (No.18540284).

## References

- [1] Particle Data Group, J. Phys. G: Nucl. Phys. **33** (2006) 1.
- [2] Y. Koide, Lett. Nuovo Cimento **34** (1982) 201; Phys. Lett. **B120**, 161 (1983); Phys. Rev. **D28**, 252 (1983).
- [3] Y. Koide, Mod. Phys. Lett. **A5**, 2319 (1990).
- [4] S. Pakvasa and H. Sugawara, Phys. Lett. **B82**, 105 (1979); Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. **D25**, 1895 (1982); P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B458**, 79 (1999); Phys. Lett. **B530**, 167 (2002); Z. Z. Xing, Phys. Lett. **B533**, 85 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. **B535**, 163 (2003); Phys. Lett. **B557**, 76 (2003); E. Ma, Phys. Rev. Lett. **90**, 221802 (2003); C. I. Low and R. R. Volkas, Phys. Rev. **D68**, 033007 (2003); X.-G. He and A. Zee, Phys. Lett. **B560**, 87 (2003).
- [5] Y. Koide and H. Fusaoka, Z. Phys. **C71**, 459 (1996).
- [6] Y. Koide and M. Tanimoto, Z. Phys. **C72**, 333 (1996).
- [7] S. Pakvasa and H. Sugawara, Phys. Lett. **B73**, 61 (1978); H. Harari, H. Haut and J. Weyers, Phys. Lett. **B78**, (1978); E. Derman, Phys. Rev. **D19** (1979) 317; 459 D. Wyler, Phys. Rev. **D19**, 330 (1979).
- [8] Y. Koide, Phys. Rev. **D60**, 077301 (1999).
- [9] Y. Koide, Phys. Rev. **D73**, 057901 (2006). Also, see Y. Koide, Phys. Rev. **D60**, 077301 (1999).
- [10] Y. Koide, arXiv: hep-ph/0701018.
- [11] E. Ma, arXiv: hep-ph/0612022.
- [12] C. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
- [13] C. Hagedorn, M. Linder and R. N. Mohapatra, JHEP, **0606**, 042 (2006) (hep-ph/0602244).
- [14] D. G. Michael *et al.* MINOS Collaboration, Phys. Rev. Lett. **97**, 191801 (2006). Also see, J. Hosaka *et al.* the Super-Kamiokande Collaboration, arXiv: hep-ex/0604011.