

Radiatively Induced Neutrino Mass Matrix in a SUSY GUT Model with R -Parity Violation

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A neutrino mass matrix model based on a SUSY GUT model is proposed, and the admissible form of the radiatively induced neutrino mass matrix is investigated. The model can evade the problems of the proton decay due to the R -parity violating interactions and of the unwelcome constraint $\sin^2 2\theta_{solar} > 0.99$ in the Zee model. The model can favorably fit to the observed neutrino data with a nearly bimaximal mixing.

1. Introduction

The Zee model [1] is one of promising models of neutrino mass generation mechanism, because the model has only 3 free parameters and it can naturally lead to a large neutrino mixing [2], especially, to a bimaximal mixing [3]. However, the original Zee model is not on a framework of a grand unification theory (GUT), and moreover, it is recently pointed out [4] that the predicted value of $\sin^2 2\theta_{solar}$ must be satisfied the relation $\sin^2 2\theta_{solar} > 0.99$ for $\Delta m_{solar}^2 / \Delta m_{atm}^2 \sim 10^{-2}$, i.e.,

$$\sin^2 2\theta_{solar} \geq 1 - \frac{1}{16} \left(\frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \right)^2. \quad (1.1)$$

The conclusion cannot be loosened even if we take the renormalization group equation (RGE) effects into consideration.

The simple ways to evade the constraint (1.1) may be as follows: One is to consider [5] that the Yukawa vertices of the charged leptons can couple to both scalars ϕ_1 and ϕ_2 . Another one [6] is to introduce a single right-handed neutrino ν_R and a second singlet Zee scalar S^+ . Also, a model with a new doubly charged scalar k^{++} is interesting because the two loop effects in such a model can give non-negligible contributions to the neutrino masses [7]. As another attractive

model, there is an idea [8] that in an R -parity violating supersymmetric (SUSY) model we identify the Zee scalar h^+ as the slepton \tilde{e}_R . Then, we can obtain additional contributions from the down-quark loop diagrams to the neutrino masses, so that such a model can be free from the constraint (1.1).

However, these models have not been embedded into a GUT scenario. As an extended Zee model based on a GUT scenario, there is, for example, the Haba-Matsuda-Tanimoto model [9]. They have regarded the Zee scalar h^+ as a member of the messenger field $M_{10} + \overline{M}_{10}$ of SUSY-breaking on the basis of an SU(5) SUSY GUT. However, their model cannot escape from the constraint (1.1) because the radiative masses are only induced by the charged lepton loop diagrams.

In the present paper, we will investigate an extended Zee model which is based on a framework of a SUSY GUT with R -parity violation, and which is free from the severe constraint (1.1). Usually, it is accepted that SUSY models with R -parity violation are incompatible with a GUT scenario, because the R -parity violating interactions induce the proton decay. In order to suppress the proton decay due to the R -parity violating terms, we will introduce a discrete symmetry Z_2 .

2. How to evade the proton decay

We identify the Zee scalar h^+ as the slepton \tilde{e}_R^+ which is a member of SU(5) 10-plet sfermions

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$\tilde{\psi}_{10}$. Then, the Zee interactions correspond to the following R -parity violating interactions

$$\begin{aligned} & \lambda_{ij}^k (\overline{\psi_{\bar{5}}^c})^A (\psi_{\bar{5}})^B (\tilde{\psi}_{10})_{kAB} \\ &= \frac{1}{\sqrt{2}} \lambda_{ij}^k \left\{ \varepsilon_{\alpha\beta\gamma} (\bar{d}_R)_i^\alpha (d_R^c)_j^\beta (\tilde{u}_R^\dagger)_k^\gamma \right. \\ & \quad - [(\bar{e}_L^c)_i (\nu_L)_j - (\bar{\nu}_L^c)_i (e_L)_j] (\tilde{e}_R^\dagger)_k \\ & \quad - [(\bar{e}_L^c)_i (d_R^c)_j^\alpha - (\bar{d}_R)_i^\alpha (e_L)_j] (\tilde{u}_L)_{k\alpha} \\ & \quad \left. + [(\bar{\nu}_L^c)_i (d_R^c)_j^\alpha - (\bar{d}_R)_i^\alpha (\nu_L)_j] (\tilde{d}_L)_{k\alpha} \right\}, \quad (2.1) \end{aligned}$$

where $\psi^c \equiv C\bar{\psi}^T$ and the indexes (i, j, \dots) , (A, B, \dots) and (α, β, \dots) are family-, $SU(5)_{GUT}$ - and $SU(3)_{color}$ -indexes, respectively. However, in GUT models, if the interactions (2.1) exist, the following R -parity violating interactions will also exist:

$$\begin{aligned} & \lambda_{ij}^k (\overline{\psi_{\bar{5}}^c})^A (\psi_{10})_{kAB} (\tilde{\psi}_{\bar{5}})^B \\ &= \frac{1}{\sqrt{2}} \lambda_{ij}^k \left\{ \varepsilon_{\alpha\beta\gamma} (\bar{d}_R)_i^\alpha (\tilde{d}_R^\dagger)_j^\beta (u_R^c)_k^\gamma \right. \\ & \quad - [(\bar{e}_L^c)_i (\tilde{\nu}_L)_j - (\bar{\nu}_L^c)_i (\tilde{e}_L)_j] (e_R^c)_k \\ & \quad - [(\bar{e}_L^c)_i (\tilde{d}_R^\dagger)_j^\alpha - (\bar{d}_R)_i^\alpha (\tilde{e}_L)_j] (u_L)_{k\alpha} \\ & \quad \left. + [(\bar{\nu}_L^c)_i (\tilde{d}_R^\dagger)_j^\alpha - (\bar{d}_R)_i^\alpha (\tilde{\nu}_L)_j] (d_L)_{k\alpha} \right\}, \quad (2.2) \end{aligned}$$

which contribute to the proton decay through the intermediate state \tilde{d}_R .

In order to forbid the contribution of the interactions (2.2) to the proton decay, for example, we can assume that the R -parity violating interactions occur only when the field ψ_{10} of the third family is related, i.e., we assume the interactions

$$\lambda_{ij}^3 (\overline{\psi_{\bar{5}}^c})^A (\psi_{10})_{3AB} (\tilde{\psi}_{\bar{5}})^B, \quad (2.3)$$

instead of the interaction (2.2). Then, the terms $\lambda_{ij}^3 (\bar{d}_R)_i (\tilde{d}_R^\dagger)_j (u_R^c)_3$ cannot contribute to the proton decay. In order to realize the constraints

$$\lambda_{12}^k = \lambda_{23}^k = \lambda_{31}^k = 0 \quad \text{for } k = 1, 2, \quad (2.4)$$

we introduce a discrete symmetry Z_2 , which exactly holds at every energy scale, as follows:

$$\begin{aligned} (\psi_{\bar{5}})_i &\rightarrow \eta_i (\psi_{\bar{5}})_i, & (\tilde{\psi}_{\bar{5}})_i &\rightarrow \eta_i (\tilde{\psi}_{\bar{5}})_i, \\ (\psi_{10})_i &\rightarrow \xi_i (\psi_{10})_i, & (\tilde{\psi}_{10})_i &\rightarrow \xi_i (\tilde{\psi}_{10})_i, \end{aligned} \quad (2.5)$$

where η_i and ξ_i take

$$\eta = (+1, +1, +1), \quad \xi = (-1, -1, +1), \quad (2.6)$$

under the Z_2 symmetry. Then, the Z_2 invariance leads to the constraints (2.4).

However, if the RGE effects cause a mixing between the first and third families, the interactions (2.3) can again contribute to the proton decay. If we assume that $\mathbf{5}$ and $\bar{\mathbf{5}}$ Higgs fields H_u and H_d transform as

$$H_u \rightarrow +H_u, \quad H_d \rightarrow +H_d, \quad (2.7)$$

under the Z_2 symmetry, the up-quark mass matrix M_u is given by the form

$$M_u = \begin{pmatrix} c_u & d_u & 0 \\ d_u & b_u & 0 \\ 0 & 0 & a_u \end{pmatrix}. \quad (2.8)$$

This guarantees that the top quark u_3 in the R -parity violating terms (2.3) does not mix with the other components (u_1 and u_2) even if we take the RGE effects into consideration, so that the interactions (2.3) cannot contribute to the proton decay at any energy scales.

On the other hand, the down-quark mass matrix M_d and the charged lepton mass matrix M_e , which are generated by the Higgs scalar H_d , have the form

$$M_d = M_e^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_1 & a_2 & a_3 \end{pmatrix}. \quad (2.9)$$

The mass matrix form (2.9) cannot explain the observed masses and mixings. For this problem, we have two options: One is to consider that we have mass generation mechanism from some higher dimensional diagram, like a mechanism proposed by Froggatt, Lowe and Nielsen [10], in addition to the mass generation (2.9) at the three level. Another one is to consider an $SU(5)$ 45-plet Higgs field H_{45} with the transformation

$$H_{45} \rightarrow -H_{45}, \quad (2.10)$$

under the Z_2 symmetry. Then, we obtain

$$M_d = \begin{pmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{pmatrix}, \quad (2.11)$$

$$M_e^T = \begin{pmatrix} -3c_1 & -3c_2 & -3c_3 \\ -3b_1 & -3b_2 & -3b_3 \\ a_1 & a_2 & a_3 \end{pmatrix}, \quad (2.12)$$

The H_{45} Higgs scalar cannot contribute to the up quark mass matrix M_u because $\bar{\psi}_{10} M_u \psi_{10}^c$ belongs to $(\overline{10} \times \overline{10})_{\text{symmetric}}$. However, in the latter option, the **45** Higgs scalar H_{45} has a vacuum expectation value (VEV) $\langle H_{45}^0 \rangle$ at the electroweak energy scale Λ_L , so that the Z_2 symmetry is broken at $\mu = \Lambda_L$. Therefore, the proton decay may occur through higher order Feynman diagrams. We suppose that such effects will be suppressed by a factor Λ_L/Λ_X (Λ_X is a unification scale). However, since the purpose of the present paper is to discuss the phenomenology of the neutrino mass matrix, for the present, we will not touch any more which option is reasonable.

3. Radiatively induced neutrino masses

We define fields u_i , d_i and e_i as those corresponding to mass eigenstates, i.e.,

$$H_{\text{mass}} = \bar{u}_L U_L^{u\dagger} M_u U_R^u u_R + \bar{d}_L U_L^{d\dagger} M_d U_R^d d_R + \bar{e}_L U_L^{e\dagger} M_e U_R^e e_R + h.c., \quad (3.1)$$

and fields ν_{Li} as partners of the mass eigenstates e_{Li} , i.e., $\ell_{Li} = (\nu_{Li}, e_{Li})$. We define the neutrino mass matrix M_ν as

$$H_{\nu \text{ mass}} = \bar{\nu}_L^c M_\nu \nu_L. \quad (3.2)$$

Therefore, a unitary matrix U_L^ν which is defined by

$$U_L^{\nu T} M_\nu U_L^\nu = D_\nu \equiv \text{diag}(m_1^\nu, m_2^\nu, m_3^\nu), \quad (3.3)$$

is identified as the Maki-Nakagawa-Sakata-Pontecorvo [11] neutrino mixing matrix $U_{MNSP} = U_\nu$.

In addition to the R -parity violating terms (2.1) and (2.2) [(2.3)], we assume SUSY breaking terms $\tilde{\psi}_{\bar{5}} \tilde{\psi}_{10} H_{\bar{5}}^d$ (and $\tilde{\psi}_{\bar{5}} \tilde{\psi}_{10} H_{45}^d$). For simplicity, we do not consider $\tilde{e}_R^c - H_d^\dagger$ mixing as in the original Zee model. Then, the neutrino masses are radiatively generated. In Fig. 1, we illustrate the Feynman diagram for the case with the down-quark loop. The amplitude is proportional to the coefficient

$$(U_R^{d\dagger} \lambda U_L^e)_{lj} (\tilde{U}_L^d)_{3n} \cdot (U_L^{d\dagger} M_d U_R^d)_{kl}$$

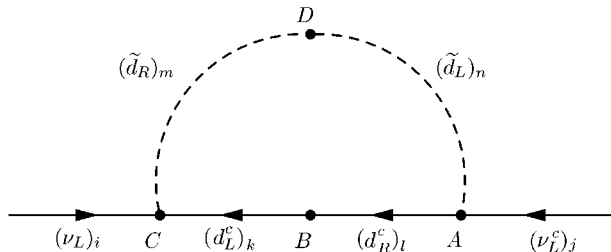


Figure 1. Radiatively induced neutrino mass through the down-quark loop. The vertexes A, B, C and D are given by $(U_R^{d\dagger} \lambda U_L^e)_{lj} (\tilde{U}_L^d)_{3n}$, $(U_L^{d\dagger} M_d U_R^d)_{kl}$, $(\tilde{U}_R^{d\dagger} \lambda U_L^e)_{mi} (U_L^d)_{3k}$, and $(\tilde{U}_L^{d\dagger} \tilde{m}_d^2 \tilde{U}_R^d)_{kl}$, respectively.

$$\begin{aligned} & \cdot (\tilde{U}_R^{d\dagger} \lambda^T U_L^e)_{mi} (U_L^d)_{3k} \cdot (\tilde{U}_L^{d\dagger} \tilde{m}_d^2 \tilde{U}_R^d)_{nm} \\ & = (\tilde{m}_d^2 \lambda U_L^e)_{3i} (M_d \lambda U_L^e)_{3j}, \end{aligned} \quad (3.4)$$

where $(\tilde{m}_d^2)_{ij}$ are coefficients of $(\tilde{d}_L^c)_i (\tilde{d}_R^c)_j$, and $(\lambda)_{ij} = \lambda_{ij}^3$. Similarly, we obtain the contributions from the charged lepton loops. Therefore, the radiatively induced neutrino mass matrix M_ν is given by the following form

$$(M_\nu)_{ij} = (f_i^e g_j^e + f_j^e g_i^e) K_e + (f_i^d g_j^d + f_j^d g_i^d) K_d, \quad (3.5)$$

where K_f ($f = e, d$) are common factors independently of the families, and

$$\begin{aligned} f_i^e &= (M_e^T \lambda U_L^e)_{3i}, & g_i^e &= (\tilde{m}_e^{2T} \lambda U_L^e)_{3i}, \\ f_i^d &= (M_d \lambda U_L^e)_{3i}, & g_i^d &= (\tilde{m}_d^2 \lambda U_L^e)_{3i}. \end{aligned} \quad (3.6)$$

Moreover, since $(M_e^T)_{3i} = (M_d)_{3i}$ as seen in (2.11) and (2.12), we obtain the relations

$$f_i^e = f_i^d \equiv f_i. \quad (3.7)$$

4. Phenomenology

Suggested from the relations (3.7), we assume that $(\tilde{m}_e^{2T})_{3i} = (\tilde{m}_d^2)_{3i}$, i.e., $g_i^e = g_i^d \equiv g_i$. Then, the neutrino mass matrix (3.5) becomes a simple form

$$(M_\nu)_{ij} = m_0 (f_i g_j + f_j g_i). \quad (4.1)$$

Hereafter, for convenience, we will normalize f_i and g_i as

$$|f_1|^2 + |f_2|^2 + |f_3|^2 = 1, \quad |g_1|^2 + |g_2|^2 + |g_3|^2 = 1. \quad (4.2)$$

In the most SUSY models, it is taken that the form of \tilde{m}_f^2 ($f = e, d$) is proportional to the fermion mass matrix M_f . Then, the coefficients g_i are proportional to f_i , so that the mass matrix (4.1) becomes $(M_\nu)_{ij} = 2m_0 f_i f_j$, which is a rank one matrix. Therefore, we rule out the case with $\tilde{m}_f^2 \propto M_f$.

For convenience, hereafter, we assume that f_i and g_i ($i = 1, 2, 3$) are real. The mass eigenvalues and mixing matrix elements for the neutrino mass matrix (4.1) are given as follows :

$$\begin{aligned} m_1^\nu &= (1 + \varepsilon)m_0, \\ m_2^\nu &= -(1 - \varepsilon)m_0, \\ m_3^\nu &= 0, \end{aligned} \quad (4.3)$$

$$\begin{aligned} U_{i1} &= \frac{1}{\sqrt{2}} \frac{f_i + g_i}{\sqrt{1 + \varepsilon}}, \\ U_{i2} &= \frac{1}{\sqrt{2}} \frac{f_i - g_i}{\sqrt{1 - \varepsilon}}, \\ U_{i3} &= -\varepsilon_{ijk} \frac{f_j g_k}{\sqrt{1 - \varepsilon^2}}, \end{aligned} \quad (4.4)$$

where

$$\varepsilon = f_1 g_1 + f_2 g_2 + f_3 g_3, \quad (4.5)$$

As seen in (4.3), the mass level pattern of the present model shows the inverse hierarchy as well as that of the Zee model. From (4.3), we obtain

$$\begin{aligned} \Delta m_{21}^2 &\equiv (m_2^\nu)^2 - (m_1^\nu)^2 = -4\varepsilon m_0^2, \\ \Delta m_{32}^2 &\equiv (m_3^\nu)^2 - (m_2^\nu)^2 = -(1 - \varepsilon)^2 m_0^2, \end{aligned} \quad (4.6)$$

$$R \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{4\varepsilon}{(1 - \varepsilon)^2}. \quad (4.7)$$

For a small R , the mixing parameters $\sin^2 2\theta_{solar}$, $\sin^2 2\theta_{atm}$ and U_{e3}^2 are given by

$$\begin{aligned} \sin^2 2\theta_{solar} &\equiv 4U_{11}^2 U_{12}^2 = \frac{1}{1 - \varepsilon^2} (f_1^2 - g_1^2)^2, \\ \sin^2 2\theta_{atm} &\equiv 4U_{23}^2 U_{33}^2 \\ &= \frac{4}{(1 - \varepsilon^2)^2} [f_2 f_3 + g_2 g_3 - \varepsilon (f_3 g_2 + f_2 g_3)]^2, \end{aligned} \quad (4.8)$$

$$U_{e3}^2 = 1 - \frac{f_1^2 + g_1^2 - 2\varepsilon f_1 g_1}{1 - \varepsilon^2}. \quad (4.10)$$

The atmospheric neutrino data [12] require $f_2 f_3 + g_2 g_3 \simeq \pm 1/2$ from (4.9). For example, we phenomenologically assume

$$\begin{aligned} f_1 &= s_\alpha, \quad f_2 = f_3 = \frac{1}{\sqrt{2}} c_\alpha, \\ g_1 &= c_\beta, \quad g_2 = g_3 = -\frac{1}{\sqrt{2}} s_\beta, \end{aligned} \quad (4.11)$$

where $c_\alpha = \cos \alpha$, $s_\alpha = \sin \alpha$ and so on. Then, the parameterization (4.11) gives

$$\varepsilon = \sin(\alpha - \beta), \quad (4.12)$$

$$\sin^2 2\theta_{solar} = \frac{1}{1 - \varepsilon^2} (c_\beta^2 - s_\alpha^2), \quad (4.13)$$

$$\sin^2 2\theta_{atm} = \frac{1}{(1 - \varepsilon^2)^2} (c_\alpha^2 + c_\beta^2 - 2\varepsilon c_\alpha s_\beta)^2, \quad (4.14)$$

$$U_{e3}^2 = 0. \quad (4.15)$$

We assume that the values of α and β are highly close each other, i.e., $\sin(\alpha - \beta) \sim 10^{-2}$. In the limit of $\alpha = \beta$, we obtain

$$\sin^2 2\theta_{solar} \simeq \cos^2 2\alpha, \quad (4.16)$$

$$\sin^2 2\theta_{atm} \simeq 1. \quad (4.17)$$

The result (4.16) is free from the constraint (1.1) in the original Zee model, so that we can fit the value of $\sin^2 2\theta_{solar}$ with the observed value [13] $\sin^2 2\theta_{solar} \sim 0.8$ from the solar neutrino data by adjusting the parameter α ($\simeq \beta$).

From the recent atmospheric and solar neutrino data [12,13]

$$R \simeq \frac{4.5 \times 10^{-5} \text{ eV}^2}{2.5 \times 10^{-3} \text{ eV}^2} = 1.8 \times 10^{-2}, \quad (4.18)$$

we estimate

$$\varepsilon = 4.5 \times 10^{-3}, \quad (4.19)$$

and

$$m_0 \simeq m_1^\nu \simeq |m_2^\nu| \simeq \sqrt{\Delta m_{atm}^2} = 0.050 \text{ eV}. \quad (4.20)$$

The effective neutrino mass $\langle m_\nu \rangle$ from the neutrinoless double beta decay experiment is given by

$$\langle m_\nu \rangle = (M_\nu)_{11} = 2m_0 c_\alpha s_\beta$$

$$\simeq m_0 \sqrt{1 - \sin^2 2\theta_{\text{solar}}} \simeq 2.2 \times 10^{-3} \text{ eV}, \quad (4.21)$$

where we have used the observed value [13] $\sin^2 2\theta_{\text{solar}} \simeq 0.8$. The value (4.21) is too small compared with the recent experimental value [14] of $\langle m_\nu \rangle$, even if we take the uncertainty of the nuclear matrix elements into consideration. If the observed value [14] $\langle m_\nu \rangle = (0.11 - 0.56) \text{ eV}$ is established, the present model will be ruled out. We hope further experimental studies of $\langle m_\nu \rangle$.

5. Conclusion

In conclusion, we have proposed a neutrino mass matrix model based on a SUSY GUT model where only top quark takes R -parity violating interactions and the Z_2 symmetry plays an essential role, so that the proton decay due to the R -parity interactions can be evaded safely. The model has four parameters, so that it can evade the constraint $\sin^2 2\theta_{\text{solar}} > 0.99$ in the Zee model. The model can favorably fit the observed atmospheric and solar neutrino data with a nearly bimaximal mixing. However, the numerical set of $(M_e)_{i3}$, $(\tilde{m}_e^2)_{i3}$ and λ_{ij}^3 , which leads to the parameterization (4.11) is not unique. What forms of M_e , M_d and λ_{ij}^3 can give the relations (4.11) together with reasonable quark and charged lepton mass matrices is our future task.

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REFERENCES

1. A. Zee, Phys. Lett. **93B** (1980) 389; **161B** (1985) 141; L. Wolfenstein, Nucl. Phys. **B175** (1980) 93; S. T. Petcov, Phys. Lett. **115B** (1982) 401.
 2. A. Yu. Smirnov and M. Tanimoto, Phys. Rev. **D55** (1997) 1665.
 3. C. Jarlskog, M. Matsuda, S. Skadhauge, M. Tanimoto, Phys. Lett. **B449** (1999) 240.
 4. Y. Koide, Phys. Rev. **D64** (2001) 077301.
- Also, see P. H. Frampton and S. L. Glashow, Phys. Lett. **B461** (1999) 95. For recent studies, see, P. H. Frampton, M. C. Oh and T. Yoshikawa, hep-ph/0110300; B. Brahmachari and S. Choubey, hep-ph/0111133.
5. K. R. S. Balaji, W. Grimus, T. Schwetz, Phys. Lett. **B508** (2001) 301.
 6. D. A. Dicus, H.-J. He and J. N. Ng, Phys. Rev. Lett. **87** (2001) 111803.
 7. A. Zee, Nucl. Phys. **264B** (1986) 99; K. S. Babu, Phys. Lett. **B203** (1988) 132; D. Chang, W.-Y. Keung and P. B. Pal, Phys. Rev. Lett. **61** (1988) 2420. For recent works, see, for examples, L. Lavoura, Phys. Rev. **D62** (2000) 093011; T. Kitabayashi and M. Yasue, Phys. Lett. **B490** (2000) 236.
 8. M. Drees, S. Pakvasa, X. Tata and T. ter Veldhuis, Phys. Rev. **D57** (1998) 5335; G. Bhattacharyya, H. V. Klapdor-Kleingrothaus and H. Pas, Phys. Lett. **B463** (1999) 77; K. Cheung and O. C. W. Kong, Phys. Rev. **D61** (2000) 113012.
 9. N. Haba, M. Matsuda and M. Tanimoto, Phys. Lett. **B478** (1999) 351.
 10. C. D. Froggatt, G. Lowe and H. B. Nielsen, Nucl. Phys. **B414** (1994) 579; C. D. Froggatt, H. B. Nielsen and J. J. Smith, Phys. Lett. **B385** (1996) 150. And also see, C. D. Froggatt, H. B. Nielsen and Y. Takanishi, hep-ph/0201152.
 11. Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870; B. Pontecorvo, Zh. Eksp. Theor. Fiz. **33** (1957) 549; Sov. Phys. JETP **26** (1968) 984.
 12. T. Toshito, Talk presented on XXXVIth Rencontres de Moriond, Electroweak Interactions and Unified Theories, 10-17 March 2001, hep-ex/0105023.
 13. J. N. Bahcall, M. C. Gonzalez-Garcia and C. Penã-Garay, JHEP **0108** (2001) 014; G. L. Fogli, E. Lizi, D. Montanino and A. Palazzo, Phys. Rev. **D64** (2001) 093007; V. Barger, D. Marfatia and K. Whisnant, hep-ph/0106207; P. I. Krastev and A. Yu. Smirnov, hep-ph/0108177.
 14. H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney and I. V. Krivosheina,

