

# New U(3) Family Gauge Symmetry and Muonium into Antimuonium Conversion <sup>1</sup>

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## Abstract

According to Sumino's idea, a family U(3) gauge symmetry is assumed. SU(2)<sub>L</sub> doublet fields  $q_L$  and  $\ell_L$  are assigned to  $\mathbf{3}$  of U(3), while singlets  $u_R, d_R, e_R$  and  $\nu_R$  are assigned to  $\mathbf{3}^*$  of U(3). Then, current-current interactions with flavor number violations of  $|\Delta N_f| = 2$  ( $N_f$  is an individual family number) appear via the family gauge boson exchanges. Since the gauge symmetry model has inevitably been brought by Sumino with a specific purpose, the gauge coupling constants  $g_f$  and the gauge boson mass spectrum  $m_{fij} \equiv m(A_i^j)$  are not free parameters. We estimate  $m_{f11} \sim 10^{0-1}$  TeV and  $m_{f12} \sim 10^{1-2}$  TeV. As a possible signature of such the flavor number violating interactions, muonium into antimuonium conversion is discussed together with a rare kaon decay  $K^+ \rightarrow \pi^+ + \mu^- + e^+$ .

## 1 Why do we need a U(3) family gauge symmetry?

### 1.1 Mystery of the charged lepton mass relation

Investigations of mass spectra have always provided promising clues for solving problems in physics: for example, the Balmer formula brought the Bohr theory, and the Gell-Mann-Okubo mass formula brought the quark model. Therefore, we may expect that investigation of the quark and lepton mass spectra and mixings will also provide a promising clue to new fundamental physics.

Meanwhile, in the charged lepton sector, we know that an empirical relation [1]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \quad (1)$$

is satisfied with the order of  $10^{-5}$  with the pole masses [2], i.e.

$$K^{pole} = \frac{2}{3} \times (0.999989 \pm 0.000014), \quad (2)$$

while it is only valid with the order of  $10^{-3}$  with the running masses, i.e.

$$K^{run}(\mu) = \frac{2}{3} \times (1.00189 \pm 0.00002), \quad (3)$$

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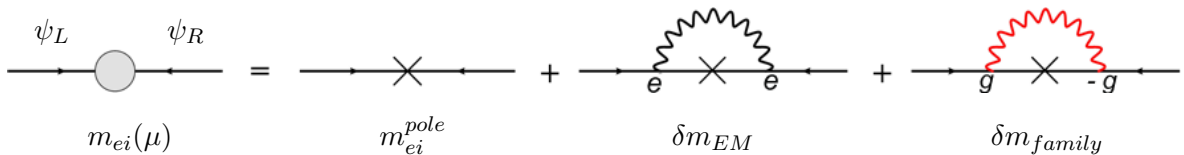


Figure 1: Radiative corrections for charged lepton masses. The diagram presents  $m_{ei}(\mu) = m_{ei}^{pole} + (\text{radiative correction due to photon}) + (\text{radiative correction due to family gauge bosons})$ .

at  $\mu = m_Z$ . In conventional mass matrix models, “mass” means not “pole mass” but “running mass.” Why is the mass formula (1) so remarkably satisfied with the pole masses? This has been a mysterious problem as to the relation (1) for long years.

## 1.2 Sumino mechanism

Recently, a possible solution of this problem has been proposed by Sumino [3, 4]. The deviation of  $K(\mu)$  from  $K^{pole}$  is caused by a logarithmic part  $m_{ei} \log m_{ei}$  in the radiative correction term [5]

$$\delta m_{EM} = -\frac{\alpha(\mu)}{\pi} m_{ei} \left( 1 + \frac{3}{4} \log \frac{\mu^2}{m_{ei}^2} \right). \quad (4)$$

(Note that the formula (1) is invariant under a transformation  $m_{ei} \rightarrow m_{ei}(1 + \varepsilon_0)$  if the value  $\varepsilon_0$  is independent of a family number.) He considers that a flavor symmetry is gauged, and the logarithmic term in the radiative correction to the charged lepton masses due to photon is exactly canceled by that due to family gauge bosons. (This does not always mean  $m_{ei}(\mu) = m_{ei}^{pole}$ .)

In order that the Sumino mechanism (cancellation mechanism) works correctly, a left-handed field  $\psi_L$  and its right-handed partner  $\psi_R$  must be assigned to  $\mathbf{3}$  and  $\mathbf{3}^*$  of U(3), respectively, i.e.  $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$ . Even apart from such a cancellation mechanism, the assignment  $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$  seems to be natural also from a point of view of grand unification (GUT) scenarios. If we adopt the conventional assignment in an SU(5) GUT model, we must consider  $(\mathbf{5}_L^*, \mathbf{3}) + (\mathbf{10}_L, \mathbf{3}^*)$  of  $SU(5) \times U(3)_{fam}$ , while we can consider  $(\mathbf{5}_L^* + \mathbf{10}_L, \mathbf{3})$  in the new assignment  $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$ . Furthermore, we can consider  $(\mathbf{16}_L, \mathbf{3})$  in an SO(10) GUT model under the new assignment.

U(3) family gauge symmetry has, for example, been proposed by Yanagida [6]. The assignment  $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$  has been proposed by Appelquist-Bai-Pia [7]. Their interests are in the possible quark and lepton mass matrix forms, and not in searches for visible effects of the gauge bosons. A new view in the present U(3) family gauge symmetry model is that the gauge coupling constants  $g_f$  and the gauge boson mass spectrum are not free parameters (see Eqs.(9) and (10) later), because the present gauge symmetry has inevitably been brought by Sumino with a specific purpose, as we give a short review in the next subsection. Since the family gauge symmetry proposed by Sumino has the energy scale  $\Lambda \sim 10^3$  TeV, at which the charged lepton mass relation  $K = 2/3$  is exactly satisfied and the gauge bosons acquire their masses as shown

in Eq.(10), differently from other family gauge symmetry models, searches for the family gauge boson effects soon become within our reach.

### 1.3 Then, what happens?

The new assignment  $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$  can induce interesting observable effects. In the conventional assignment, a family gauge boson  $A_j^i$  couples to a current component  $(J_\mu)_i^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} + \bar{\psi}_R^j \gamma_\mu \psi_{Ri}$ , while in the present model, the gauge boson  $A_j^i$  couples to

$$(J_\mu)_i^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} - \bar{\psi}_{Ri} \gamma_\mu \psi_R^j. \quad (5)$$

In general, the current-current interactions by this type of currents cause interactions which violate the individual family number  $N_f$  by  $|\Delta N_f| = 2$ . The influence of the flavor number violation is determined by the family gauge coupling constant  $g_f$  and each family gauge boson mass  $m_{fij} \equiv m(A_i^j)$ . Here, for simplicity, the flavor current structure has been presented only by a field  $\psi$  as a representative of the realistic fields (quarks  $u_i$  and  $d_i$  and leptons  $e_i$  and  $\nu_i$ ). For example, the current component  $(J_\rho)_1^2$  is given by

$$(J_\rho)_1^2 = \bar{\mu}_L \gamma_\rho e_L - \bar{e}_R \gamma_\rho \mu_R. \quad (6)$$

This causes a  $\mu$  lepton number violation process  $e^- + e^- \rightarrow \mu^- + \mu^-$  through the effective current-current interaction

$$\mathcal{L}^{eff} = \frac{G_{f12}}{\sqrt{2}} [\bar{\mu} \gamma_\rho (1 - \gamma_5) e] [\bar{e} \gamma^\rho (1 + \gamma_5) \mu] + h.c., \quad (7)$$

where

$$\frac{G_{f12}}{\sqrt{2}} = \frac{g_f^2}{8(m_{f12})^2}. \quad (8)$$

( $m_{f12} = m(A_2^1)$ ). Also we may expect flavor violation processes with  $|\Delta N_f| = 2$  ( $N_f$  is an individual family number) such as  $e^- + e^- \rightarrow \mu^- + \mu^-$ ,  $u + e^- \rightarrow c + \mu^-$ ,  $u + u \rightarrow t + t$ , and so on.

In the conventional models with  $|\Delta N_f| = 2$  [e.g. the dilepton model [8]],  $g_f$  and  $m_{f12}$  are free parameters. In contrast to the conventional models, the present model has the following constraints on the parameters:

(i) The coupling constant  $g_f$  is related to the electric charge  $e$  as

$$\frac{1}{4} g_f^2 = e^2 = g_2^2 \sin^2 \theta_W, \quad (9)$$

( $g_2$  is the  $SU(2)_L$  gauge coupling constant) in order to work the Sumino mechanism correctly.

(ii) The gauge boson masses  $m_{fij}$  are related to the charged lepton masses  $m_{ei}$  as

$$m_{fij} \equiv m(A_i^j) \propto \sqrt{m_{ei} + m_{ej}}. \quad (10)$$

The condition (ii) comes from the following reason: In this model, Yukawa coupling constants  $Y_e^{eff}$  of the charged leptons are effectively given by

$$(Y_e^{eff})_{ij} = \frac{1}{\Lambda^2} \sum_{a=1}^3 \langle (\Phi_e)_{ia} \rangle \langle (\Phi_e^T)_{aj} \rangle, \quad (11)$$

where  $\Phi_e$  is a scalar with  $(\mathbf{3}, \mathbf{3})$  of  $U(3) \times O(3)$  family symmetries. (Here, the family  $U(3) \times O(3)$  symmetries are originated in a  $U(9)$  flavor symmetry [4], and only  $U(3)$  gauge symmetry can contribute to the radiative correction of the running masses of charged leptons below  $\Lambda \sim 10^3$  TeV, at which the charged lepton mass relation (1) is given exactly.) In other words, the VEV matrix  $\langle \Phi_e \rangle$  is given as

$$\langle \Phi_e \rangle = \text{diag}(v_1, v_2, v_3) \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}). \quad (12)$$

[A prototype of such an idea for the charged lepton masses is found in Ref. [9] related to the mass formula (1).] Then, the gauge symmetry  $U(3)$  is completely broken by  $\langle \Phi_e \rangle \neq 0$ , so that the gauge boson masses  $m_{fij}$  are related to the charged lepton masses  $m_{ei}$ . In the Sumino model, since the energy scale of the effective theory is assumed as  $\Lambda \sim 10^3$  TeV, we suppose  $m_{f13} \sim 10^{2-3}$  TeV and  $m_{f12} \sim 10^{1-2}$  TeV.

It is convenient to describe our predictions in terms of a factor  $G_{f12}/G_F$ , where  $G_F$  is the Fermi constant  $G_F/\sqrt{2} = g_2^2/8m_W^2$ :

$$\frac{G_{fij}}{G_F} = 4 \sin^2 \theta_W \left( \frac{m_W}{m_{fij}} \right)^2 = \frac{5.98 \times 10^{-3}}{(m_{fij} [\text{TeV}])^2}. \quad (13)$$

## 2 Gauge boson mass constraints from rare kaon decays

### 2.1 Current form in the down-quark sector

Next we discuss rare kaon decays. Note that, in the present model, the family number  $i = (1, 2, 3)$  is defined by  $i = (e, \mu, \tau)$  in the charged lepton sector. If we assume  $i = (1, 2, 3) \sim (d, s, b)$  for the down-quark sector, the gauge boson masses  $m_{f12}$  can be constrained by the rare kaon decay searches.

In general, the family gauge current forms in the quark sectors depend on quark flavor mixing matrices which are given in the diagonal basis of the charged lepton mass matrix  $M_e$ . For simplicity, we assume that  $M_d$  is Hermitian and consider only a  $d$ - $s$  mixing

$$\begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix} = U_d \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (14)$$

where the down-quark mass matrix  $M_d$  is given in the flavor basis in which the charged lepton mass matrix  $M_e$  is diagonal, and  $M_d$  is diagonalized as  $U_d^\dagger M_d U_d = \text{diag}(m_d, m_s, m_b)$ . In this

case, the down-quark current  $(J_\mu^{(d)})_1^2$  is given by

$$(J_\mu^{(d)})_1^2 = \bar{s}_L^0 \gamma_\mu d_L^0 - \bar{d}_R^0 \gamma_\mu s_R^0 = \frac{1}{2}(\bar{s} \gamma_\mu d - \bar{d} \gamma_\mu s) - \frac{1}{2}(\bar{s} \gamma_\mu \gamma_5 d + \bar{d} \gamma_\mu \gamma_5 s) \cos 2\theta + \frac{1}{2}(\bar{s} \gamma_\mu \gamma_5 s - \bar{d} \gamma_\mu \gamma_5 d) \sin 2\theta, \quad (15)$$

where the first, second and third terms have  $CP = -1, +1$  and  $+1$ , respectively. Note that the vector current is independent of the mixing angle  $\theta$ . (However, this is valid only in the case  $M_d^\dagger = M_d$ .)

As an example of the  $s$ - $d$  current, let us discuss a decay of neutral kaon into  $e^\pm + \mu^\mp$ . In Eq. (15), only the second term is relevant to a neutral kaon with spin-parity  $0^-$ , which has  $CP = +1$ . Thus, we must identify the second term as  $K_S$  (not  $K_L$ ) in the limit of  $CP$  conservation. Hence, a stringent lower limit of  $m_{f12}$  cannot be extracted from the present experimental limit [2]  $Br(K_L \rightarrow e^\pm \mu^\mp) < 4.7 \times 10^{-12}$ .

## 2.2 Constraints on the gauge boson mass $m_{f12}$

Instead, the lower limit of  $m_{f12}$  can be obtained from the rare kaon decays  $K^+ \rightarrow \pi^+ + e^\pm + \mu^\mp$ . The  $K \rightarrow \pi$  decay is described by the first term (vector currents) in Eq. (15), which can be replaced by  $i(\pi^- \overleftrightarrow{\partial}_\rho K^+)$ .

$$\begin{aligned} \mathcal{L}^{eff} &= \frac{g_f^2}{2m_f^2} (\bar{s}_L \gamma_\rho d_L - \bar{d}_R \gamma_\rho s_R + \bar{\mu}_L \gamma_\rho e_L - \bar{e}_R \gamma_\rho \mu_R) (\bar{d}_L \gamma^\rho s_L - \bar{s}_R \gamma^\rho d_R + \bar{e}_L \gamma^\rho \mu_L - \bar{\mu}_R \gamma^\rho e_R) \\ &\Rightarrow 2 \frac{G_{f12}}{\sqrt{2}} (\bar{s} \gamma_\rho d) (\bar{e} \gamma^\rho \mu - \bar{\mu} \gamma^\rho e) \Rightarrow 2 \frac{G_{f12}}{\sqrt{2}} i(\pi^- \overleftrightarrow{\partial}_\rho K^+) (\bar{e} \gamma^\rho \mu - \bar{\mu} \gamma^\rho e). \end{aligned} \quad (16)$$

Since the effective interaction for  $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$  is given by

$$\mathcal{L}_{weak} = \frac{g_2^2}{2m_W^2} V_{us} (\bar{s}_L \gamma_\rho u_L) (\bar{\mu}_L \gamma^\rho \nu_{\mu L}), \quad (17)$$

the ratio  $Br(K^+ \rightarrow \pi^+ e^\pm \mu^\mp) / Br(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu)$  is given by

$$R = \frac{[2 \cdot (G_{f12}/\sqrt{2})]^2}{2|V_{us}|^2 (1/\sqrt{2})^2 (G_F/\sqrt{2})^2} = 67.27 \left( \frac{m_W}{m_{f12}} \right)^4, \quad (18)$$

in the approximation  $m(\pi^+) = m(\pi^0)$  and  $m(e^-) = m(\nu_\mu) = 0$ . Here, we have used the relation (9) for coupling constant  $g_f$ . Thus, the ratio (18) is given only as a linear function of the ratio  $(m_W/m_{f12})^4$ .

The present experimental limits [2]

$$Br(K^+ \rightarrow \pi^+ e^- \mu^+) < 1.3 \times 10^{-11}, \quad Br(K^+ \rightarrow \pi^+ \mu^- e^+) < 5.2 \times 10^{-10}, \quad (19)$$

Table 1: Lower bounds of family gauge boson mass  $m_{f12}$  estimated from the rare decays  $K^+ \rightarrow \pi^+ e^\pm \mu^\mp$ . Here, we have used  $Br(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu) = (3.35 \pm 0.04) \times 10^{-2}$ .

Decay mode	$K^+ \rightarrow \pi^+ \mu^- e^+$	$K^+ \rightarrow \pi^+ e^- \mu^+$
Experiments	$Br < 5.2 \times 10^{-10}$	$Br < 1.3 \times 10^{-11}$
Lower bound	$m_{f12} > 21$ TeV	$m_{f12} > 52$ TeV

Table 2: Masses of the gauge bosons  $A_1^1$ ,  $A_2^1$ ,  $A_3^1$  and  $A_3^3$ , and their lower bounds from rare kaon decays. Their relative sizes are also shown.

	$m_{f11}$	$m_{f12}$	$m_{f13}$	$m_{f33}$
Relative sizes	$\sqrt{2m_e}$	$\sqrt{m_\mu + m_e}$	$\sqrt{m_\tau + m_e}$	$\sqrt{2m_\tau}$
	0.0981127	1.00000	4.09154	5.78448
$K^+ \rightarrow \pi^+ \mu^- e^+$	2.1 TeV	21 TeV	86 TeV	120 TeV
$K^+ \rightarrow \pi^+ e^- \mu^+$	5.1 TeV	52 TeV	210 TeV	300 TeV

together with  $Br(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu) = (3.35 \pm 0.04) \times 10^{-2}$  give lower limits of the gauge boson mass  $m_{f12}$  as shown in Table 1. Note that the mode  $K^+ \rightarrow \pi^+ e^+ \mu^-$  has  $|\Delta N_f| = 2$ , which we are interested in, while the mode  $K^+ \rightarrow \pi^+ e^- \mu^+$  has  $|\Delta N_f| = 0$ .

We can estimate lower bounds of other gauge boson masses,  $m_{f11}$ ,  $m_{f13}$ , etc., from the lower bounds of  $m_{f12}$  using the relation (10). The results are listed in Table 2. In the present model, the energy scale of the effective theory  $\Lambda$  has been required as  $\Lambda \sim 10^3$  TeV. On the other hand, as seen in Table 2, the lower bound of the mass  $m_{f33}$  of the heaviest gauge boson  $A_3^3$  is 300 TeV from  $K^+ \rightarrow \pi^+ e^- \mu^+$ . Therefore, the lower bound of each gauge boson listed in Table 2 seems to be almost near to its upper bound. In other words, the mass values given in Table 2 suggest that experimental observations of family gauge boson effects soon become within our reach.

### 3 Muonium into antimuonium conversion

#### 3.1 Why important to investigate muonium-antimuonium conversion

Exactly speaking, the constraints from kaon rare decays may depend on the down-quark mixing structure and some another hadronic effects, although we suppose that such effects are small. We would like to test for the new gauge boson effects in the pure leptonic processes. So, the investigation of muonium  $M(\mu^+ e^-)$  into antimuonium  $\bar{M}(\mu^- e^+)$  conversion is indispensable.

#### 3.2 Conversion probability

The constraint on the gauge boson mass  $m_{f12}$  is also obtained from a muonium into antimuonium conversion  $M(\mu^+ e^-) \rightarrow \bar{M}(\mu^- e^+)$ . The total  $M\bar{M}$  conversion probability  $P_{M\bar{M}}(B)$  under an

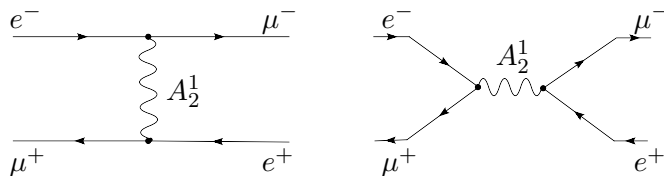


Figure 2: Diagram of muonium into antimuonium conversion

external magnetic field  $B$  is generally given by

$$P_{M\bar{M}}(B) = \frac{\delta^2}{2[\delta^2 + (E_M - E_{\bar{M}})^2 + \lambda^2]}, \quad (20)$$

where a mass matrix  $M_{mass}$  for the state  $(M, \bar{M})$  is given by

$$M_{mass} = \begin{pmatrix} E_M & \frac{1}{2}\delta \\ \frac{1}{2}\delta & E_{\bar{M}} \end{pmatrix}, \quad (21)$$

$E_M$  and  $E_{\bar{M}}$  are energies of muonium  $M$  and antimuonium  $\bar{M}$ ,  $\lambda$  is the muon decay width, and  $\delta$  is defined by  $\langle \bar{M} | H_{M\bar{M}} | M \rangle$  which is proportional to  $(G_{f12}/\sqrt{2})/\pi a^3$  ( $a$  is the Bohr radius). The case with the effective interaction (7) of a type  $(V - A)(V + A)$  is effectively equivalent to a case in the dilepton model [8], and the formulation has been investigated by Horikawa and Sasaki [11] in details: A case with  $(V - A)(V + A)$  interaction predicts

$$P_{M\bar{M}}(0) \simeq \frac{3}{2} \frac{\delta^2}{\lambda^2}, \quad (22)$$

and

$$\delta = -8 \frac{G_{f12}}{\sqrt{2}} \frac{1}{\pi a^3}, \quad (23)$$

so that we obtain

$$P_{M\bar{M}}(0) = 1.96 \times 10^{-5} \times \left( \frac{G_{f12}}{G_F} \right)^2 = 7.01 \times 10^{-10} \frac{1}{m_{f12}} [\text{TeV}]^4. \quad (24)$$

In Table 3, we list predicted values of the predict muonium into antimuonium conversion probability  $P_{M\bar{M}}(0)$  for specific values of the family gauge boson mass  $m_{f12}$  which correspond to the lower limits from rare kaon decays  $K^+ \rightarrow \pi^+ e^\mp \mu^\pm$ . Also, we illustrate behavior of  $P_{M\bar{M}}(0)$  versus  $m_{f12}$  in Fig.3.

### 3.3 Present experimental limit on the gauge boson mass $m_{f12}$

Present experimental limit [12] of the total conversion probability integrated over all decay times  $P_{M\bar{M}}$  is

$$P_{M\bar{M}}(B) \leq 8.3 \times 10^{-11} \quad (90\% \text{CL}) \quad \text{in } B = 0.1 \text{ T}. \quad (25)$$

Table 3: Muonium into antimuonium conversion probability  $P_{M\bar{M}}(0)$  for specific values of the family gauge boson mass  $m_{f12}$  which correspond to the lower limits from rare kaon decays  $K^+ \rightarrow \pi^+ e^\mp \mu^\pm$ .

Input: $m_{f12}$	21 TeV	52 TeV
Prediction: $P_{M\bar{M}}(0)$	$3.6 \times 10^{-15}$	$9.6 \times 10^{-17}$

Since  $S_B(0.1T) = 0.78$  for the case of  $(V - A)(V + A)$  [11], where  $S_B(B)$  is defined by

$$P_{M\bar{M}}(B) = P_{M\bar{M}}(0)S_B(B), \quad (26)$$

this bound leads to  $P_{M\bar{M}}(0) \leq 1.06 \times 10^{-10}$ , and to  $G_{f12}/G_F \leq 2.3 \times 10^{-3}$ , so that the lower bound of the mass  $m_{f12}$  is given by

$$m_{f12} \geq 19.9 m_W = 1.60 \text{ TeV}. \quad (27)$$

The constraint (27) is looser than that from the  $K^+$  decay given in Table 1. However, it should be noticed that the values in given in Table 1 are dependent on the down-quark mixing matrices  $U_{dL}$  and  $U_{dR}$  which are unknown at present differently from the CKM mixing  $V = U_{uL}^\dagger U_{dL}$ . We would like to emphasize that observations in the pure leptonic processes are also important independently of the bounds from the rare kaon decays. We expect that future experiments improve this bound (27).

## 4 Concluding remarks

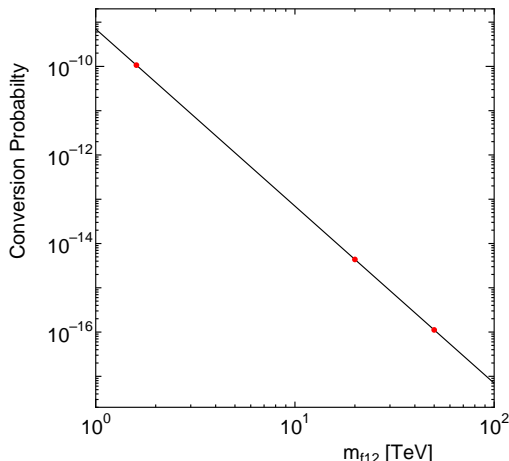


Figure 3: Conversion probability  $P_{M\bar{M}}(0)$  versus a gauge boson mass  $m_{f12}$ . Points from the left to the right correspond to the lower limits of the gauge boson mass  $m_{f12}$ , 1.6 TeV, 21 TeV and 52 TeV, which have been estimated from the muonium into antimuonium conversion, rare kaon decays  $K^+ \rightarrow \pi^+ \mu^- e^+$  and  $K^+ \rightarrow \pi^+ e^- \mu^+$ , respectively.

Although the present lower limit of  $m_{f12}$  from  $M$ - $\bar{M}$  conversion experiment is considerably looser than those from rare kaon decays as shown in Fig.3. However, I again would like to emphasize that, in the present model, the family number  $i = (1, 2, 3)$  is defined by  $i = (e, \mu, \tau)$  in the charged lepton sector. Only when we assume  $i = (e, \mu, \tau) \sim (d, s, b)$  for the down-quark sector, the gauge boson masses  $m_{f12}$  can be constrained by the rare kaon decay searches as given

in Table 2. Therefore, the test for the present family gauge symmetry in the  $M-\bar{M}$  conversion is still indispensable. Besides, if we find a positive evidence in the  $M-\bar{M}$  conversion, and if we find that the mass  $m_{f12}$  is lower than the lower limit of  $m_{f12}$  from the kaon decays, we can obtain an important clue to the down-quark mixing  $U_d$  (not  $V_{CKM}$ !) at the flavor basis in which the charged lepton mass matrix takes a diagonal form. We expect that we will soon find a positive evidence in the  $M-\bar{M}$  conversion.

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