

Tests of New Family Gauge Symmetry

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(Dated: July 27, 2010)

We explore the structure of a new family gauge symmetry $U(3)$ and show its experimental signatures to search for. $U(3)$ gauge bosons obviate an unwelcome deviation of the charged lepton mass formula with the running masses from that with the pole masses. The current structure of this model leads to flavor number violations via exchange of extra gauge bosons. We obtain bounds on the masses of the gauge bosons from rare kaon decay searches and muonium-antimuonium oscillation searches. We propose attractive signatures at LHC and lepton colliders and discuss feasibility of their discovery.

PACS numbers: 12.60.-i, 14.70.Pw, 11.30.Hv, 13.66.-a,

Why do we need a new family symmetry? – For a unified understanding of quarks and leptons, the observed mass spectra and flavor mixings will provide a promising clue. In the charged lepton sector, we know that an empirical relation [1]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \quad (1)$$

is satisfied with the order of 10^{-5} with the pole masses, i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ [2], while it is only valid with the order of 10^{-3} with the running masses, i.e. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$. In conventional mass matrix models, “mass” means not “pole mass” but “running mass.” Why is the mass formula (1) so remarkably satisfied with the pole masses? This has been a mysterious problem as to the relation (1) for long years.

Recently, a possible solution to this problem has been proposed by one of the authors (Y.S.) [3, 4]: The deviation of $K(\mu)$ from K^{pole} is caused by a logarithmic term $m_{ei} \log(\mu/m_{ei})$ in the running mass. It was advocated that a family symmetry is gauged, and that the logarithmic term in the radiative correction to $K(\mu)$ due to photon is exactly canceled by that due to family gauge bosons. (This does not mean $m_{ei}(\mu) = m_{ei}^{pole}$.) In order that cancellation works correctly, the left-handed lepton field ψ_L and its right-handed partner ψ_R should be assigned to $\mathbf{3}$ and $\mathbf{3}^*$ of $U(3)$, respectively, differently from the conventional assignment $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3})$ [5].

Even apart from such a cancellation mechanism, the assignment $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$ seems to be natural also from a viewpoint of grand unification (GUT) scenarios. If we adopt the conventional assignment in an $SU(5)$ GUT model, we must consider $(\mathbf{5}_L^*, \mathbf{3}) + (\mathbf{10}_L, \mathbf{3}^*)$ of $SU(5) \times U(3)_{fam}$, while we can consider $(\mathbf{5}_L^* + \mathbf{10}_L, \mathbf{3})$ in the new assignment $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$. Furthermore, we can consider $(\mathbf{16}_L, \mathbf{3})$ in an $SO(10)$ GUT model under the new assignment.

The new assignment $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$ can induce in-

teresting observable effects. In the conventional assignment, a family gauge boson A_j^i couples to a current component $(J_\mu)_i^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} + \bar{\psi}_R^j \gamma_\mu \psi_{Ri}$, while in the present model, the gauge boson A_j^i couples to

$$(J_\mu)_i^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} - \bar{\psi}_{Ri} \gamma_\mu \psi_R^j. \quad (2)$$

In general, the current-current interactions by this type of currents cause interactions which violate the individual family number N_f by $|\Delta N_f| = 2$. The influence of the flavor number violation is determined by the family gauge coupling constant g_f and each family gauge boson mass $m_{fij} \equiv m(A_i^j)$. In order to realize the cancellation mechanism [3] between photon and family gauge bosons, g_f is related to the electric charge e as

$$\frac{1}{4} g_f^2 = e^2 \equiv g_2^2 \sin^2 \theta_W, \quad (3)$$

(g_2 is the gauge coupling constant of $SU(2)_L$), so that the ratio of the coefficients of the four-Fermi contact interactions is given by

$$\frac{G_{fij}}{G_F} = 4 \sin^2 \theta_W \left(\frac{m_W}{m_{fij}} \right)^2 = \frac{5.98 \times 10^{-3}}{(m_{fij} [\text{TeV}])^2}. \quad (4)$$

Here $G_{fij}/\sqrt{2} = g_f^2/8m_{fij}^2$ and $G_F/\sqrt{2} = g_2^2/8m_W^2$.

In this model, Yukawa coupling constants Y_e^{eff} of the charged leptons are effectively given by

$$(Y_e^{eff})_{ij} = \frac{1}{\Lambda^2} \sum_{a=1}^3 \langle (\Phi_e)_{ia} \rangle \langle (\Phi_e^T)_{aj} \rangle, \quad (5)$$

where Φ_e is a scalar with $(\mathbf{3}, \mathbf{3})$ of family $U(3) \times O(3)$ symmetries. In other words, the VEV matrix $\langle \Phi_e \rangle$ is given as $\langle \Phi_e \rangle = \text{diag}(v_1, v_2, v_3) \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$. [A prototype of such an idea for the charged lepton masses is found in Ref. [6] related to the mass formula (1).] Then, the gauge symmetry $U(3)$ is completely broken

by $\langle \Phi_e \rangle \neq 0$, so that the gauge boson masses m_{fij} are related to the charged lepton masses as [4]

$$(m_{fij})^2 \equiv m^2(A_i^j) \propto m_{ei} + m_{ej}. \quad (6)$$

In this model [4], it is speculated that the cut-off scale Λ of the effective theory is of the order of 10^{2-3} TeV, and the masses of A_i^j are in the 10^{1-3} TeV range.

The purpose of the present paper is to discuss how to test this new family gauge symmetry. We examine the interactions with $|\Delta N_f| = 2$ via the gauge boson A_2^1 . We estimate a lower bound of its mass m_{f12} from the experimental limit on the branching ratio of a rare kaon decay $K^+ \rightarrow \pi^+ \mu^- e^+$. We also discuss muonium into antimuonium conversion and $e^- + e^- \rightarrow \mu^- + \mu^-$ production. (For a review of searches for signatures with $|\Delta N_f| = 2$, see, for example, Ref.[7].) Furthermore, a search for the gauge boson A_1^1 , which is the lightest one of the family U(3) gauge bosons, is considered. We presume that A_1^1 has a mass of 10^{1-2} TeV, according to the aforementioned model. We may expect a production $p + p \rightarrow A_1^1 + X \rightarrow (e^+ e^-) + X$ at LHC, for which we investigate the cross section and decay rate.

Characteristic structure of the effective current-current interactions – First, in order to see more details of the characteristic current structure (2), we discuss the flavor changing neutral currents relevant for μ and e . By Eq. (2), the current can be written as

$$(J_\rho)_1^2 = \bar{\mu}_L \gamma_\rho e_L - \bar{e}_R \gamma_\rho \mu_R = (J_V)_\rho - (J_A)_\rho, \quad (7)$$

where $(J_V)_\rho = (1/2)(\bar{\mu} \gamma_\rho e - \bar{e} \gamma_\rho \mu)$ and $(J_A)_\rho = (1/2)(\bar{\mu} \gamma_\rho \gamma_5 e + \bar{e} \gamma_\rho \gamma_5 \mu)$. The vector current J_V^ρ and axial current J_A^ρ have $CP = -1$ and $CP = +1$, respectively. However, this does not mean that the effective current-current interactions cause CP -violating interactions. In fact, the current $(J_\rho)_2^1$ is written as

$$(J^\rho)_2^1 = \bar{e}_L \gamma^\rho \mu_L - \bar{\mu}_R \gamma^\rho e_R = -(J_V)^\rho - (J_A)^\rho. \quad (8)$$

Hence, the effective current-current interaction is CP conserving:

$$\begin{aligned} \mathcal{L}^{eff} &= 4 \frac{G_{f12}}{\sqrt{2}} (J_\rho)_1^2 (J^\rho)_2^1 \\ &= -4 \frac{G_{f12}}{\sqrt{2}} \left[(J_V)_\rho (J_V)^\rho - (J_A)_\rho (J_A)^\rho \right]. \end{aligned} \quad (9)$$

Rare kaon decays – Next we discuss rare kaon decays. In general, a down-quark mass matrix M_d is not necessarily diagonal in the diagonal basis of the charged lepton mass matrix M_e . For simplicity, we assume that M_d is Hermitian and consider only a d - s mixing ($d^0 = d \cos \theta - s \sin \theta$, $s^0 = d \sin \theta + s \cos \theta$). In this case,

the down-quark current $(J_\mu^{(d)})_1^2$ is given by

$$\begin{aligned} (J_\mu^{(d)})_1^2 &= \bar{s}_L^0 \gamma_\mu d_L^0 - \bar{d}_R^0 \gamma_\mu s_R^0 \\ &= \frac{1}{2} (\bar{s} \gamma_\mu d - \bar{d} \gamma_\mu s) - \frac{1}{2} (\bar{s} \gamma_\mu \gamma_5 d + \bar{d} \gamma_\mu \gamma_5 s) \cos 2\theta \\ &\quad + \frac{1}{2} (\bar{s} \gamma_\mu \gamma_5 s - \bar{d} \gamma_\mu \gamma_5 d) \sin 2\theta, \end{aligned} \quad (10)$$

where the first, second and third terms have $CP = -1$, $+1$ and $+1$, respectively. Note that the vector current is independent of the mixing angle θ . (However, this is valid only in the case $M_d^\dagger = M_d$.)

As an example of the s - d current, let us discuss a decay of neutral kaon into $e^\pm + \mu^\mp$. In Eq. (10), only the second term is relevant to a neutral kaon with spin-parity 0^- , which has $CP = +1$. Thus, we must identify the term as K_S (not K_L) in the limit of CP conservation. Hence, a stringent lower limit of m_{f12} cannot be extracted from the present experimental limit [2] $BR(K_L \rightarrow e^\pm \mu^\mp) < 4.7 \times 10^{-12}$.

Instead, the lower limit of m_{f12} can be obtained from the rare kaon decays $K^+ \rightarrow \pi^+ + e^\pm + \mu^\mp$. The $K \rightarrow \pi$ decay is described by the first term (vector currents) in Eq. (10), which can be replaced by $i(\pi^- \overleftrightarrow{\partial}_\rho K^+)$. Hence,

$$\begin{aligned} \mathcal{L}^{eff} &= 2(G_{f12}/\sqrt{2})(\bar{s} \gamma_\rho d)(\bar{e} \gamma^\rho \mu - \bar{\mu} \gamma^\rho e) \\ &\Rightarrow 2(G_{f12}/\sqrt{2})i(\pi^- \overleftrightarrow{\partial}_\rho K^+)(\bar{e} \gamma^\rho \mu - \bar{\mu} \gamma^\rho e). \end{aligned} \quad (11)$$

Since the effective interaction for $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ is given by $\mathcal{L}_{weak} = (g_2^2/2m_W^2)V_{us}(\bar{s}_L \gamma_\rho u_L)(\mu_L \gamma^\rho \nu_{\mu L})$, the ratio $BR(K^+ \rightarrow \pi^+ e^\pm \mu^\mp)/BR(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu)$ is given by

$$R = \frac{[2 \cdot (G_{f12}/\sqrt{2})]^2}{2|V_{us}|^2(1/\sqrt{2})^2(G_F/\sqrt{2})^2} = 67.27 \left(\frac{m_W}{m_{f12}} \right)^4, \quad (12)$$

in the approximation $m(\pi^+) = m(\pi^0)$ and $m(e^-) = m(\nu_\mu) = 0$. The present experimental limits [2] $BR(K^+ \rightarrow \pi^+ e^- \mu^+) < 1.3 \times 10^{-11}$ and $BR(K^+ \rightarrow \pi^+ \mu^- e^+) < 5.2 \times 10^{-10}$ together with $BR(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu) = (3.35 \pm 0.04) \times 10^{-2}$ give lower limits of the gauge boson mass m_{f12} as shown in Table I. Note that the mode $K^+ \rightarrow \pi^+ e^+ \mu^-$ has $|\Delta N_f| = 2$, which we are interested in, while the mode $K^+ \rightarrow \pi^+ e^- \mu^+$ has $|\Delta N_f| = 0$. We can estimate lower bounds of other gauge boson masses, m_{f11} , m_{f13} , etc., from the lower bounds of m_{f12} using the relation (6). The results are listed in Table I. Since we presume $m_{f12} \sim 10^{1-2}$ TeV, the values given in Table I suggest that experimental observations of family gauge boson effects soon become within our reach.

Muonium-antimuonium conversion – A constraint on the gauge boson mass m_{f12} is also obtained from a muonium into antimuonium conversion $M(\mu^+ e^-) \rightarrow \bar{M}(\mu^- e^+)$. The total $M\bar{M}$ conversion probability $P_{M\bar{M}}(B)$ under an external magnetic field B is given by $P_{M\bar{M}}(B) = \delta^2/2[\delta^2 + (E_M - E_{\bar{M}})^2 + \lambda^2]$, where E_M

TABLE I: Masses of the gauge bosons A_1^1 , A_2^1 , A_3^1 and A_3^3 , and their lower bounds from rare kaon decays. Their relative sizes are also shown.

	m_{f11}	m_{f12}	m_{f13}	m_{f33}
Relative sizes	$\sqrt{2}m_e$	$\sqrt{m_\mu + m_e}$	$\sqrt{m_\tau + m_e}$	$\sqrt{2}m_\tau$
	0.0981127	1.00000	4.09154	5.78448
$K^+ \rightarrow \pi^+ \mu^- e^+$	2.1 TeV	21 TeV	86 TeV	120 TeV
$K^+ \rightarrow \pi^+ e^- \mu^+$	5.1 TeV	52 TeV	210 TeV	300 TeV

and $E_{\overline{M}}$ are the energies of M and \overline{M} , respectively, λ is the bound muon decay width, and δ is defined by $\langle M | H_{M\overline{M}} | \overline{M} \rangle$ which is proportional to $(G_{f12}/\sqrt{2})/\pi a^3$ (a is the electron Bohr radius). We derive the effective interaction describing $M\overline{M}$ conversion from (9),

$$\mathcal{L}_{\Delta N_f=2}^{eff} = \frac{G_{f12}}{\sqrt{2}} [\bar{\mu}(1 - \gamma^5)e] [\bar{\mu}(1 + \gamma^5)e]. \quad (13)$$

This has the same $(V - A)(V + A)$ form as the one corresponding to a dilepton model [8], and the formulation in this case has been investigated by Horikawa and Sasaki [9] in detail. It predicts $P_{M\overline{M}}(0) \simeq (3/2)\delta^2/\lambda^2$ and $\delta = -8(G_{f12}/\sqrt{2})(1/\pi a^3)$. It follows that

$$P_{M\overline{M}}(0) = 1.96 \times 10^{-5} \times \left(\frac{G_{f12}}{G_F} \right)^2 = \frac{7.01 \times 10^{-10}}{(m_{f12} [\text{TeV}])^4}. \quad (14)$$

For example, for $m_{f12} = 21$ TeV and 52 TeV, Eq. (14) predicts $P_{M\overline{M}}(0) = 3.6 \times 10^{-15}$ and 9.6×10^{-17} , respectively. Present experimental limit [10] of the total conversion probability integrated over all decay times is $P_{M\overline{M}}(B) \leq 8.3 \times 10^{-11}$ (90% CL) for $B = 0.1$ T. Since $S_B(0.1\text{T}) = 0.78$ for the case of $(V - A)(V + A)$ [9], this bound leads to $P_{M\overline{M}}(0) \leq 1.06 \times 10^{-10}$, and to $G_{f12}/G_F \leq 2.3 \times 10^{-3}$. Thus, the lower bound of m_{f12} is given by

$$m_{f12} \geq 20 m_W = 1.6 \text{ TeV}. \quad (15)$$

This constraint is looser than those from the K^+ decays. However, it should be noted that the values in Table I are dependent on the down-quark mixing matrices U_{dL} and U_{dR} (if $M_d^\dagger \neq M_d$), which are unknown at present apart from the CKM matrix $V = U_{uL}^\dagger U_{dL}$. We would like to emphasize that observations in the pure leptonic processes are important independently of the bounds from the rare kaon decays. We expect that future experiments will improve the bound Eq. (15).

Production with $|\Delta N_f| = 2$ – Next, we investigate possible signatures of the current-current interaction with $|\Delta N_f| = 2$ at collider experiments. Although a top-top production at LHC (via $u + u \rightarrow t + t$) is very attractive, the cross section $\sim 10^{-6}$ pb at $\sqrt{s} = 14$ TeV and

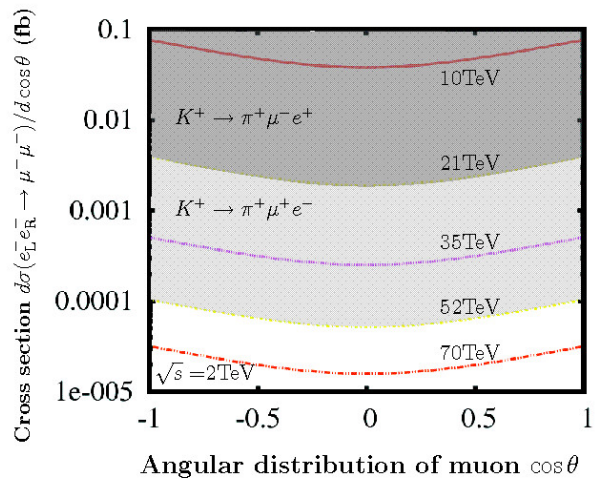


FIG. 1: Differential cross section $d\sigma(e_L^- e_R^- \rightarrow \mu^- \mu^-)/d \cos \theta$ vs. $\cos \theta$. We set $\sqrt{s} = 2$ TeV and $m_{f12} = 10, 35,$ and 70 TeV. The light-shaded (dark-shaded) region is excluded by the rare decay $K^+ \rightarrow \pi^+ e^- \mu^+$ ($K^+ \rightarrow \pi^+ e^+ \mu^-$), as shown in Table I.

for $m_{f13} = 10^2$ TeV would be too small to detect the signal. The cross section for $e^- + p \rightarrow \mu^- + X$ amounts to $\sigma \sim 10^{-5}$ pb at $E_p = 7$ TeV and $E_e = 400$ GeV for $m_{f12} = 50$ TeV, which would also be difficult to detect, because of a large background $e^- + p \rightarrow \mu^- + \nu_e + \bar{\nu}_\mu + p$ with $\sigma \sim 10^{-1}$ pb.

The most clean reaction with $|\Delta N_f| = 2$ is $e^- + e^- \rightarrow \mu^- + \mu^-$. This reaction is expected at an optional experiment at a future $e^+ e^-$ linear collider. The current structure in this model shows that this reaction takes place only between invertedly polarized electron pairs $e_L^- e_R^-$. This aspect is useful for discriminating this model from others using the polarized e^- beams. We obtain the differential cross section

$$\frac{d\sigma}{d \cos \theta} = \frac{2\pi\alpha_{\text{EM}}^2}{m_{f12}^4} s(1 + \cos^2 \theta), \quad (16)$$

and the total cross section $\sigma(e_L^- e_R^- \rightarrow \mu^- \mu^-) = (16\pi\alpha_{\text{EM}}^2/3m_{f12}^4)s$. Figure 1 shows the differential cross sections $d\sigma(e_L^- e_R^- \rightarrow \mu^- \mu^-)/d \cos \theta$ at the c.m. energy $\sqrt{s} = 2$ TeV. The value of the family gauge boson mass m_{f12} corresponding to each line is displayed in the figure. For $m_{f12} = 21$ (52) TeV and at $\sqrt{s} = 2$ TeV, the total cross section is given by $\sigma = 3.3 \times 10^{-2}$ (8.7×10^{-4}) fb. A high luminosity operation of a future lepton collider may lead to the model confirmation by observing the clean reaction with $|\Delta N_f| = 2$.

Search for the lightest gauge boson A_1^1 – Finally, we discuss a search for the gauge boson A_1^1 , which is the lightest one of the family U(3) gauge bosons. For simplicity, we neglect the up-quark mixing. The method is practically the same as that for Z' boson. (For reviews of Z' , see, for instance, Refs. [11]. In particular, the highest limit of Z' mass from direct searches is about 1 TeV, which

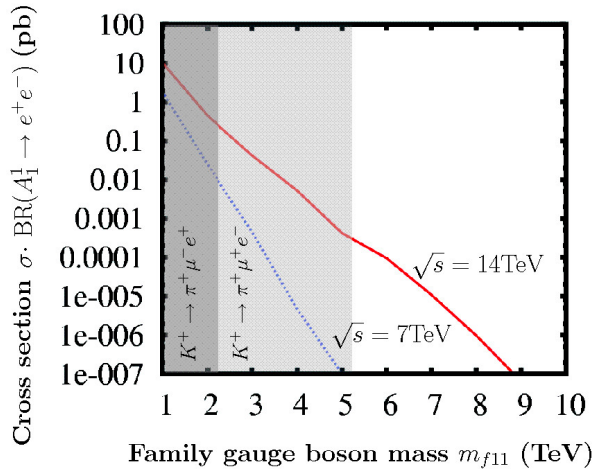


FIG. 2: $\sigma(pp \rightarrow A_1^1 X) \cdot \text{BR}(A_1^1 \rightarrow e^+e^-)$ as a function of the family gauge boson mass m_{f11} . The light-shaded (dark-shaded) region is excluded, as noted in Fig. 1.

TABLE II: Cross sections for the signal and Drell-Yan background, and S/\sqrt{N} corresponding to an integrated luminosity of 1 fb^{-1} , at LHC $\sqrt{s} = 14 \text{ TeV}$. No cuts are imposed.

$m_{f11}(\text{TeV})$	signal (fb)	DY BG (fb)	S/\sqrt{N}
2	4.4×10^2	1.6×10^{-1}	1.1×10^3
3	4.2×10	1.5×10^{-2}	3.4×10^2

is much smaller than the bounds on m_{f11} in Table I.) In conventional Z' models, Z' couples to fermions of all flavors, whereas the A_1^1 boson couples only to the first generation, i.e., $A_1^1 \rightarrow e^+e^-, \nu_e\bar{\nu}_e, u\bar{u}, d\bar{d}$. The total decay width and the branching ratio are given, respectively, by

$$\begin{aligned} \Gamma(A_1^1 \rightarrow \text{all}) &= (5/16\pi)g_f^2 m_{f11} = 5\alpha_{em}m_{f11}, \\ \text{BR}(A_1^1 \rightarrow e^+e^-) &= 2/15, \end{aligned} \quad (17)$$

which are different from those of conventional Z' models. Since we presume that A_1^1 has a mass of 10^{1-2} TeV , it is impossible to find A_1^1 at Tevatron. On the other hand, we may expect productions of A_1^1 at LHC. In Fig.2, we show the cross section $\sigma(pp \rightarrow A_1^1 X \rightarrow e^+e^- X) = \sigma(pp \rightarrow A_1^1 X) \cdot \text{BR}(A_1^1 \rightarrow e^+e^-)$ for $\sqrt{s} = 7 \text{ TeV}$ and 14 TeV . The cross sections are calculated with CalcHEP [12] implementing Eq. (2) and with the CTEQ6L code [13] for the parton distribution function. When we reconstruct dilepton invariant masses $m(l^+l^-)$, if we observe a peak in $m(e^+e^-)$ but no peak in $m(\mu^+\mu^-)$, this will be a signal of the new gauge boson A_1^1 . (This feature is unchanged even with up-quark mixing.)

The dominant backgrounds in the A_1^1 search, after moderate event selection cuts, are Drell-Yan dielectrons [14]. Table II lists S/\sqrt{N} as a measure of A_1^1 discovery reach for $m_{f11} \leq 3 \text{ TeV}$. Estimates of backgrounds within

a window of $\pm 4\Gamma_{Z'} \approx \pm \Gamma_{A_1^1}$ before any cut are taken from [14]. Comparing to the analysis given there, we anticipate that, with an integrated luminosity of 10 fb^{-1} , m_{f11} up to several TeV would be within discovery reach. However, we leave a detailed study to our future work.

Summary – So far, introduction of the new family gauge symmetry is the only known explanation for $K(\mu) = K^{pole}$. Therefore, tests of the model are urgently required. Although the gauge boson masses m_{fij} are constrained by the rare kaon decay searches as shown in Table I, the results should not be taken rigidly, since we do not know the structure of down-quark mixing (not the CKM mixing) in the diagonal basis of the charged lepton mass matrix. (In general, the lower bounds in Table I may be lowered.) A quark-mixing independent bound is obtained from a purely leptonic process, muonium-antimuonium conversion, and more sensitive tests will come from an upgrade of this experiment or from the process $e_L^- e_R^- \rightarrow \mu^- \mu^-$ at ILC. Moreover, we may expect to observe a peak in $m(e^+e^-)$ but no peak in $m(\mu^+\mu^-)$ at LHC. We hope that these searches will make a breakthrough in new physics discoveries.

Acknowledgments – The authors would like to thank Y. Kuno for fruitful discussion. Y.S. is grateful to H. Yokoya for useful discussion. The works of Y.K. and Y.S. are supported by JSPS (Nos. 21540266 and 20540246, respectively). The work of M.Y. is supported by Maskawa Institute in Kyoto Sangyo University.

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