

Yukawaon Model in the Quark Sector and Nearly Tribimaximal Neutrino Mixing

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Abstract

For the purpose of deriving the observed nearly tribimaximal neutrino mixing, a possible yukawaon model in the quark sector is investigated. Five observable quantities (2 up-quark mass ratios and 3 neutrino mixing parameters $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{solar}$ and $|U_{13}|$) are excellently fitted by two parameters (one in the up-quark sector and another one in the right-handed Majorana neutrino sector).

Keywords: neutrino mixing matrix, CKM mixing matrix, yukawaon model

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1 Introduction

1.1 Why do we investigate a quark mass matrix model for the purpose of discussing the neutrino mixing?

Recently, on the basis of the so-called ‘‘yukawaon’’ model [1, 2] (we will give a short review in the next subsection), the author has proposed a curious neutrino mass matrix form [3]: although the mass matrix M_ν is given by a conventional seesaw-type form $M_\nu = m_D M_R^{-1} m_D^T$, the Majorana mass matrix M_R of the right-handed neutrinos ν_R is related to a up-quark mass matrix M_u as

$$M_R \propto M_u^{1/2} M_e + M_e M_u^{1/2}, \quad (1.1)$$

and the neutrino Dirac mass matrix m_D is given by $m_D \propto M_e$, where M_e is the charged lepton mass matrix and $M_u^{1/2}$ is defined by $\langle M_u^{1/2} \rangle_u = \text{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$ on the diagonal basis of M_u . [Here and hereafter, we denote a form of a matrix A on the diagonal basis of M_f (we refer the basis as f basis) as $\langle A \rangle_f$.] Therefore, the mass matrix M_ν on the e basis (the diagonal basis of M_e) is given by

$$\langle M_\nu \rangle_e \propto \langle M_e \rangle_e \left\{ \langle M_u^{1/2} \rangle_e \langle M_e \rangle_e + \langle M_e \rangle_e \langle M_u^{1/2} \rangle_e \right\}^{-1} \langle M_e \rangle_e. \quad (1.2)$$

In order to calculate the lepton mixing matrix U , we must to know a form of $\langle M_u^{1/2} \rangle_e$. Since we do not know it at present, in Ref.[3], on the analogy of a relation $\langle M_u \rangle_d = V^T(\delta) \langle M_u \rangle_u V(\delta)$, where $V(\delta)$ is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix in the standard expression [4] and the observed value δ in the quark sector is $\delta \simeq 70^\circ$ [5], we have assumed

$$\langle M_u^{1/2} \rangle_e = V^T(\pi) \langle M_u^{1/2} \rangle_u V(\pi). \quad (1.3)$$

(Since we assume an $O(3)$ flavor symmetry, the mass matrices M_f must be symmetric, so that the diagonalization is done by $U_f^T M_f U_f = M_f^{diag} \equiv \langle M_f \rangle_f$.) Then, by using the observed up-quark mass ratios and CKM mixing parameters, we can obtain the lepton mixing matrix U . Usually, the so-called tribimaximal mixing [6] is understood based on discrete symmetries, while, in this model, the neutrino mass matrix (1.2) can give a nearly tribimaximal mixing $\sin^2 2\theta_{atm} = 0.995$, $\tan^2 \theta_{solar} = 0.553$ and $|U_{13}| = 0.001$ [3] without assuming any discrete symmetry. Note that the matrix V in Eq.(1.3) is not $V(\delta)$ with $\delta \simeq 70^\circ$, and it must be $V(\pi)$ in order to obtain such the successful results. This successful results rely on the ad hoc assumption (1.3). We have no theoretical ground for the ansatz (1.3).

However, if we give a quark mass matrix model in which quark mass matrices (M_u, M_d) is described on the e basis, by using U_u obtained from the diagonalization $U_u^T \langle M_u \rangle_e U_u = \langle M_u \rangle_u$, we can calculate the form of $\langle M_u^{1/2} \rangle_e$ as $\langle M_u^{1/2} \rangle_e = U_u \langle M_u^{1/2} \rangle_u U_u^T$. The purpose of the present paper is to investigate a possible quark mass matrix model which can give reasonable neutrino mixing parameters on the basis of a yukawaon model.

In the present paper, we will propose a quark mass matrix model which is described on the e basis (a diagonal basis of the charged lepton mass matrix):

$$\begin{aligned} \langle M_u^{1/2} \rangle_e &\propto \langle M_e^{1/2} \rangle_e (X + a_u \mathbf{1}) \langle M_e^{1/2} \rangle_e, \\ \langle M_d \rangle_e &\propto \langle M_e^{1/2} \rangle_e (X + a_d e^{i\alpha_d} \mathbf{1}) \langle M_e^{1/2} \rangle_e, \end{aligned} \quad (1.4)$$

where

$$X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (1.5)$$

(Such a form (1.4) has first been proposed in the context of the so-called ‘‘democratic seesaw mass matrix model [7].) As we see in the next section, the up-quark mass matrix (1.4) can give not only reasonable up-quark mass ratios at $a_u \simeq -0.56$, but also reasonable neutrino mixing parameters $\sin^2 2\theta_{atm} \simeq 1$, $\tan^2 \theta_{solar} \simeq 1/2$ and $|U_{13}| \ll 1$.

1.2 What is the yukawaon model?

Prior to investigating a quark mass matrix model, let us give a short review of the so-called yukawaon model: We regard the Yukawa coupling constants in the standard model as ‘‘effective’’ coupling constants Y_f^{eff} ($f = e, \nu, u, d$) in an effective theory, and we consider that Y_f^{eff} originate in vacuum expectation values (VEVs) of new gauge singlet scalars Y_f , i.e.

$$Y_f^{eff} = \frac{y_f}{\Lambda} \langle Y_f \rangle, \quad (1.6)$$

where Λ is a scale of the effective theory. We refer the fields Y_f as ‘‘yukawaons’’ [1, 2] hereafter. Note that in the yukawaon model, Higgs scalars are the same as ones in the conventional model, i.e. we consider only two Higgs scalars H_u and H_d as an origin of the masses (not as an origin of the mass spectra).

The Froggatt-Nielsen model [8] is well known as a model which describes masses and mixings by a VEV value of a scalar ϕ : The hierarchical structure of the masses is explained by a multiplicative structure $(\langle\phi\rangle/\Lambda)^n$ under a U(1) flavor symmetry. In contrast to the Froggatt-Nielsen model, in the yukawaon model, the scalars Y_f have 3×3 flavor components, so that they are described, for example, as $\mathbf{3} \times \mathbf{3}^* = \mathbf{1} + \mathbf{8}$ of U(3) flavor symmetry or $(\mathbf{3} \times \mathbf{3})_S = \mathbf{1} + \mathbf{5}$ of O(3) flavor symmetry. The hierarchical structures of the quark and lepton masses are understood from hierarchical eigenvalues of $\langle Y_f \rangle$, not from the multiplicative structure $(\langle Y_f \rangle/\Lambda)^n$. In order to obtain such a hierarchical structure, we must build a model with $\det\langle Y_e \rangle / (\text{Tr}[\langle Y_e \rangle]^3) \ll 1$ (for an example, see Ref.[9]). Hierarchical structures of $\langle Y_f \rangle$ in other sectors will be caused by the hierarchical structure of $\langle Y_e \rangle$.

In the present paper, we assume an O(3) flavor symmetry, so that would-be Yukawa interactions are given by [3]

$$\begin{aligned}
H_Y = & \sum_{i,j} \frac{y_u}{\Lambda} u_i^c(Y_u)_{ij} q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} d_i^c(Y_d)_{ij} q_j H_d \\
& + \sum_{i,j} \frac{y_\nu}{\Lambda} \ell_i(Y_\nu)_{ij} \nu_j^c H_u + \sum_{i,j} \frac{y_e}{\Lambda} \ell_i(Y_e)_{ij} e_j^c H_d + h.c. + \sum_{i,j} y_R \nu_i^c(Y_R)_{ij} \nu_j^c, \quad (1.7)
\end{aligned}$$

where q and ℓ are $SU(2)_L$ doublet fields, and f^c ($f = u, d, e, \nu$) are $SU(2)_L$ singlet fields. All of the yukawaons Y_f ($f = u, d, \nu, e, R$) belong to $(\mathbf{3} \times \mathbf{3})_S = \mathbf{1} + \mathbf{5}$ of O(3). In the definition (1.7) of Y_f , we can define diagonalization of the VEV matrices $\langle Y_f \rangle$ ($f = u, d, e$) as $U_f^T \langle Y_f \rangle U_f \propto \text{diag}(m_{f1}, m_{f2}, m_{f3})$, and that of the seesaw-type neutrino mass matrix $M_\nu \propto \langle Y_\nu \rangle^T \langle Y_R \rangle^{-1} \langle Y_\nu \rangle$ as $U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu1}, m_{\nu2}, m_{\nu3})$, so that we can express the quark mixing matrix V and lepton mixing matrix U as $V = U_u^\dagger U_d$, and $U = U_e^\dagger U_\nu$, respectively. In the interactions (1.7), in order to distinguish each Y_f from others, we have assumed a $U(1)_X$ symmetry in addition to the O(3) flavor symmetry, and we have assigned $U(1)_X$ charges (“sector” charges, not “flavor” charges) as $Q_X(f^c) = -x_f$, $Q_X(Y_f) = +x_f$ and $Q_X(Y_R) = 2x_\nu$. The $SU(2)_L$ doublet fields q , ℓ , H_u and H_d are assigned as sector charges $Q_X = 0$. (For the right-handed neutrinos ν^c , we assume $Q_X(\nu^c) = Q_X(e^c)$, so that the yukawaon Y_e can couple to the Dirac neutrino sector $\ell_i \nu_j^c$ as well as $\ell_i e_j^c$, and we can build a model without Y_ν [10].)

The O(3) flavor symmetry is broken at the energy scale $\mu = \Lambda$, so that all the yukawaons Y_f have VEVs with the order of $\mu = \Lambda$. (We consider that all components v_{fi} of $\langle Y_f \rangle_f = \text{diag}(v_{f1}, v_{f2}, v_{f3})$ simultaneously have non-zero values at the same energy scale Λ in spite of the hierarchical structure $|v_{f3}|^2 \gg |v_{f2}|^2 \gg |v_{f1}|^2$.) A naive estimate of Λ leads to $m_\nu \sim \langle H_u^0 \rangle^2 / \Lambda$, so that we consider $\mu = \Lambda \sim 10^{15}$ GeV. Since the O(3) flavor symmetry is completely broken at $\mu = \Lambda$, the effective coupling constants $Y_f^{eff} = (y_f/\Lambda)\langle Y_f \rangle$ evolve as in the standard model below the scale Λ .

In a supersymmetric (SUSY) yukawaon model, VEV structures of the yukawaons are obtained from SUSY vacuum conditions for a superpotential W . As a result, a VEV structure of $\langle Y_f \rangle$ in an f sector is described in terms of other yukawaon VEV matrices. In other words, a VEV structure of $\langle Y_f \rangle$ is given by observed mass matrices in other sectors. (The relation (1.1) is one of the examples.) This is just a characteristic feature in the yukawaon approach.

For example, in the present scenario, we have assumed superpotential terms

$$W_e = \mu_e [Y_e \Theta_e] + \lambda_e [\Phi_e \Phi_e \Theta_e], \quad (1.8)$$

and, from the SUSY vacuum conditions $\partial W / \partial \Theta_e = 0$, we obtain a bilinear mass relation

$$\langle Y_e \rangle = -\frac{\lambda_e}{\mu_e} \langle \Phi_e \rangle \langle \Phi_e \rangle. \quad (1.9)$$

Thus, the mass matrix $M_e^{1/2}$ in Eq.(1.2) is interpreted as $\langle \Phi_e \rangle$ in the yukawaon model. We call Φ_e as ‘‘ur-yukawaon’’, because the VEV of Φ_e plays a role in giving VEV spectrum of Y_e . Here and hereafter, for convenience, we denote $\text{Tr}[A]$ as $[A]$ simply. (We can show $\langle \Theta_e \rangle = 0$ by solving SUSY vacuum conditions for all fields simultaneously. Hereafter, we will use a character Θ for a field whose VEV is $\langle \Theta \rangle = 0$.) Also, we consider the existence of an ‘‘ur-yukawaon’’ Φ_u which has a relation $\langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle$ similar to (1.8).

The relation (1.1) is naively translated to a relation $\langle Y_R \rangle \propto \langle \Phi_u \rangle \langle Y_e \rangle + \langle Y_e \rangle \langle \Phi_u \rangle$ in the yukawaon model, but, as stated in the next section, the relation (1.1) will be slightly modified (a term with same $U(1)_X$ charge will be added with an additional parameter ξ). As a result, as we see in Sec.2, we can excellently fit 5 observable quantities (2 up-quark mass ratios and 3 neutrino mixing parameters) by the two parameters a_u and ξ . Considering such the phenomenological success of the model, in Sec.3, a possible yukawaon model in the quark sector and neutrino sector is discussed. Finally, Sec.4 is devoted to a summary and concluding remarks.

In the present model, 2 down-quark mass ratios and 4 CKM mixing parameters are described by the remaining 2 parameters a_d and α_d after a_u was fixed by the observed up-quark mass ratios. As seen in Sec.2, the results are still unsatisfactory. We need a further improvement for down-quark sector. In the present paper, we do not discuss any improvement in the down-quark sector.

2 Phenomenological study

In the present section, we give numerical studies based on phenomenological mass matrices (1.2) and (1.4). Since the aim of the present paper is not to obtain the best fit values of the parameters in the model, and it is to see a main framework of the model qualitatively, an energy scale used in the present evaluations is not required to be so rigorous. Exactly speaking, VEV relations in the present model are valid at $\mu = \Lambda \sim 10^{15}$ GeV. Since the $O(3)$ flavor symmetry is completely broken at $\mu = \Lambda$, the effective coupling constants $Y_f^{eff} = (y_f / \Lambda) \langle Y_f \rangle$ evolve as in the standard model below the scale Λ . We know that the quark mass ratios are not so sensitive to the energy scale [11]. As seen in (2.1) and (2.6) below, the observed quark mass values [11] still have large errors and, besides, they are highly dependent on the value of $\tan \beta$ in the SUSY model. Since our concern is in the quark mixing matrix $V = U_u^\dagger U_d$ and lepton mixing matrix $U = U_e^\dagger U_\nu$, for simplicity, we evaluate the mass matrices (1.4) at the energy scale $\mu = m_Z$, so that numerical results in the present section should not be taken too strictly. In the present model, quark mass matrices are described in terms of the charged lepton masses, so that, for numerical estimates, we will also use values of the running masses $m_e(\mu)$, $m_\mu(\mu)$ and $m_\tau(\mu)$ at

$\mu = m_Z$. Although the yukawaon model [2] was first proposed with the aim of understanding the well-known charged lepton mass relation [12], in the present paper, we do not adopt such a yukawaon model in the charged lepton sector. We will use the observed values [11] of the running charged lepton masses as input values, which do not satisfy the charged lepton mass relation.

First, we search for a value of the parameter a_u which can give reasonable predicted values for the up-quark mass ratios [11] $m_u/m_c = 0.0021_{-0.0008}^{+0.0013}$ ($0.0021_{-0.0009}^{+0.0012}$) and $m_c/m_t = 0.0036_{-0.0005}^{+0.0006}$ ($0.0026_{-0.0006}^{+0.0007}$) at $\mu = m_Z$ (at $\mu = \Lambda_{GUT} = 2 \times 10^{16}$ GeV with $\tan \beta = 10$). Although the ratio m_u/m_t is affected by renormalization group equation (RGE) effects, the effects are not so essential in this rough estimations in the present paper. Therefore, we will use values at $\mu = m_Z$:

$$\sqrt{m_u/m_c} = 0.045_{-0.010}^{+0.013}, \quad \sqrt{m_c/m_t} = 0.060 \pm 0.005. \quad (2.1)$$

We find that predicted values of the up-quark mass ratios at $a_u \simeq -0.56$ is in favor of the observed up-quark mass ratios (2.1): $v_{u1}/v_{u2} = (-0.0355, -0.0425, -0.0514)$ and $v_{u2}/v_{u3} = (-0.0654, -0.0570, -0.0495)$ for $a_u = (-0.55, -0.56, -0.57)$. However, if we naively regard $\langle \Phi_u \rangle_e$ given in Eq.(1.4) as $\langle M_u^{1/2} \rangle_e$ in Eq.(1.2), we cannot obtain favorable values of the neutrino mixing parameters for any values of a_u ,

Note that signs of the eigenvalues (v_{u1}, v_{u2}, v_{u3}) of $\langle \Phi_u \rangle$ are $(+, -, +)$ for the parameter value $a_u \sim -0.56$, while, in Ref.[3], we have used positive values for $\langle M_u^{1/2} \rangle_e = \text{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$. Therefore, by introducing an O(3) $\mathbf{1} + \mathbf{5}$ field P_u whose VEV is given by

$$\langle P_u \rangle_u = \mu_p \text{diag}(+1, -1, +1), \quad (2.2)$$

we can express $\langle M_u^{1/2} \rangle$ as $\langle M_u^{1/2} \rangle_u \propto \Phi_u \langle P_u \rangle_u$, so that the relation (1.2) is given by

$$\langle Y_R \rangle_e \propto \langle Y_e \rangle_e \langle P_u \rangle_e \langle \Phi_u \rangle_e + \langle \Phi_u \rangle_e \langle P_u \rangle_e \langle Y_e \rangle_e, \quad (2.3)$$

where $\langle \Phi_u \rangle_e$ and $\langle P_u \rangle_e$ are given by

$$\langle \Phi_u \rangle_e = U_u \langle \Phi_u \rangle_u U_u^T, \quad \langle P_u \rangle_e = U_u \langle P_u \rangle_u U_u^T, \quad (2.4)$$

respectively. However, when we assign U(1)_X charges which satisfy $Q_X(Y_R) = Q_X(Y_e) + Q_X(\Phi_u) + Q_X(P_u)$, we must also take terms $P_u Y_e \Phi_u + \Phi_u Y_e P_u$ into consideration as well as the terms $Y_e P_u \Phi_u + \Phi_u P_u Y_e$, because they have the same U(1)_X charges. Thus, the relation (2.3) must be modified as

$$Y_R \propto Y_e P_u \Phi_u + \Phi_u P_u Y_e + \xi (P_u Y_e \Phi_u + \Phi_u Y_e P_u). \quad (2.5)$$

We find that predicted value of $\tan^2 \theta_{solar}$ is highly dependent on the parameter ξ , although the values $\sin^2 2\theta_{atm}$ and $|U_{13}|$ are not sensitive to ξ as far as $|\xi|$ is small. We show ξ -dependence of the neutrino mixing parameters in Table 1. As seen in Table 1, the cases $\xi = +0.0005$ and $\xi = -0.0012$ can excellently fit the observed values $\sin^2 2\theta_{atm} = 1.00_{-0.13}$ [13] and $\tan^2 \theta_{solar} = 0.469_{-0.041}^{+0.047}$ [14]. At present, the reason why the parameter value of ξ is so small is unknown. This may be explain by a hidden symmetry which is unbroken in the terms without ξ , but is broken in those with ξ . This is an open question at present.

Table 1: ξ -dependence of neutrino mixing parameters. The parameter a_u is fixed at $a_u \sim -0.56$.

ξ	$\sin^2 2\theta_{atm}$	$\tan^2 \theta_{solar}$	$ U_{13} $
0	0.9848	0.7033	0.0128
+0.004	0.9825	0.4891	0.0123
+0.005	0.9819	0.4486	0.0122
+0.006	0.9812	0.4123	0.0120
-0.0011	0.9897	0.4854	0.0142
-0.0012	0.9900	0.4408	0.0143
-0.0013	0.9904	0.4008	0.0144

Table 2: CKM mixing parameters versus (a_d, α_d)

a_u	a_d	α_d	$ m_{d1}/m_{d2} $	$ m_{d2}/m_{d3} $	$ V_{us} $	$ V_{cb} $	$ V_{ub} $	$ V_{td} $
-0.56	-0.620	4°	0.1078	0.0273	0.2035	0.0666	0.0101	0.0178
-0.56	-0.625	6°	0.0783	0.0313	0.2187	0.0818	0.0123	0.0190
-0.56	-0.630	8°	0.0542	0.0362	0.2222	0.0977	0.0146	0.0194
-0.58	-0.630	2°	0.1959	0.0195	0.2272	0.0448	0.0088	0.0163

Next, we calculate down-quark mass ratios and CKM matrix parameters for the model (1.4). The observed running down-quark mass ratios [11] are

$$\frac{m_d}{m_s} = \frac{0.053_{-0.029}^{+0.051}}{(0.054_{-0.030}^{+0.058})}, \quad \frac{m_s}{m_b} = \frac{0.019 \pm 0.006}{(0.017_{-0.005}^{+0.006})}, \quad (2.6)$$

at $\mu = m_Z$ (at $\mu = \Lambda_{GUT} = 2 \times 10^{16}$ GeV with $\tan\beta = 10$) and the observed CKM mixing parameters [5] are $|V_{us}| = 0.2255 \pm 0.0019$, $|V_{cb}| = 0.0412 \pm 0.0011$, $|V_{ub}| = 0.00393 \pm 0.00036$ and $|V_{td}| = 0.0081 \pm 0.025$. In Table 2, we demonstrate predicted values of the CKM mixing parameters versus (a_d, α_d) . Here, we have taken $a_u = -0.56$ which can give reasonable up-quark mass ratios.

As seen in Table 2, the case with $a_u = -0.56$ and $(a_d, \alpha_d) = (-0.63, 8^\circ)$ can roughly give reasonable values of the down-quark mass ratios and $|V_{us}|$, but $|V_{cb}|$, $|V_{ub}|$ and $|V_{td}|$ are considerably larger than the observed values. For reference, we list a case of $a_u = -0.58$ and $(a_d, \alpha_d) = (-0.63, 2^\circ)$ in Table 3. The case can give reasonable values of $|V_{us}|$ and $|V_{cb}|$, but the predicted quark mass ratios are in poor agreement with experiments. The present model will need a further improvement, as far as the down-quark sector is concerned. However, from the qualitative point of view, it worthwhile noticing that the model can roughly predict not only the quark mass ratios but also the CKM mixing parameters and neutrino oscillation parameters by using only the 4 input parameters a_u , ξ and $a_d e^{i\alpha_d}$.

3 Yukawaons in the quark sector

In the previous section, we have obtained successful predictions on the basis of the phe-

nomenological mass matrices (1.4) and (2.5). For the phenomenological mass matrix (2.5), we assume the following superpotential terms

$$W_R = \mu_R[Y_R\Theta_R] + \frac{\lambda_R}{\Lambda} \{[(Y_e P_u \Phi_u + \Phi_u P_u Y_e)\Theta_R] + \xi[(P_u Y_e \Phi_u + \Phi_u Y_e P_u)\Theta_R]\}. \quad (3.1)$$

Here, since the VEV matrix $\langle P_u \rangle$ is diagonal in the u basis (a diagonal basis of $\langle Y_u \rangle$) as given in Eq.(2.2), the VEV matrix $\langle P_u \rangle$ must be commutable with $\langle \Phi_u \rangle$. Moreover, in order to give the form (2.2), since $[\langle P_u \rangle] = \mu_p$, $\frac{1}{2}([\langle P_u \rangle]^2 - [\langle P_u \rangle \langle P_u \rangle]) = -\mu_p^2$ and $\det \langle P_u \rangle = -\mu_p^3$, the VEV matrix $\langle P_u \rangle$ must satisfy a cubic equation

$$\langle P_u \rangle^3 - \mu_p \langle P_u \rangle^2 - \mu_p^2 \langle P_u \rangle + \mu_p^3 \mathbf{1} = 0. \quad (3.2)$$

Therefore, we assume superpotential terms for P_u

$$W_P = \lambda_P[(\Phi_u P_u - P_u \Phi_u)\Theta_P] + \varepsilon_{SB} \frac{\lambda'_P}{\Lambda} [(P_u P_u P_u - \mu_p P_u P_u - \mu_p^2 P_u + \mu_p^3)\Theta'_P]. \quad (3.3)$$

Note that the second term in (3.3) cannot conserve the $U(1)_X$ charges. (The superpotential terms (3.3) can conserve the R symmetry, because the fields Θ_P and Θ'_P are assigned as R charges $R = 2$, while Φ_u and P_u are assigned as $R = 0$.) We assume that such $U(1)_X$ symmetry breaking terms are suppressed by a small factor ε_{SB} .

On the other hand, superpotential in the quark sector is given as follows:

$$W_q = \mu_u^X [\Phi_u \Theta_u^X] + \mu_d^X [Y_d \Theta_d^X] + \sum_{q=u,d} \frac{\xi_q}{\Lambda} [\Phi_e (\Phi_{Xq} + a_q E_q) \Phi_e \Theta_q^X], \quad (3.4)$$

where the parameter a_u is real, but a_d is complex. Here the VEV matrices $\langle \Phi_{Xq} \rangle_e$ and $\langle E_q \rangle_e$ are given by

$$\langle \Phi_{Xq} \rangle_e \propto X, \quad \langle E_q \rangle_e \propto \mathbf{1}, \quad (3.5)$$

and the factor $(\Phi_{Xq} + a_q E_q)$ plays a role in breaking the $O(3)$ flavor symmetry into a discrete symmetry S_3 at the e basis. If we regard Φ_{Xq} and E_q as $\Phi_{Xu} = \Phi_{Xd} \equiv \Phi_X$ and $E_u = E_d \equiv E$, we will obtain an unwelcome relation $Q_X(\Phi_u) = Q_X(Y_d)$. Therefore, we must distinguish (Φ_{Xu}, E_u) from (Φ_{Xd}, E_d) . However, if we replace μ_u^X and μ_d^X in (3.4) with $[Y_d]$ and $[\Phi_u]$, respectively, we can regard Φ_{Xq} and E_q as $\Phi_{Xu} = \Phi_{Xd} \equiv \Phi_X$ and $E_u = E_d \equiv E$ without considering two sets (Φ_{Xu}, E_u) and (Φ_{Xd}, E_d) . Superpotential terms for the field Φ_X are given by

$$W_X = \varepsilon_{SB} \left\{ \frac{\xi_X}{\Lambda} [\Phi_X \Phi_X \Phi_X \Theta_X] + \lambda_X [\Phi_X \Phi_X \Theta_X] \right\}, \quad (3.6)$$

with $(\lambda_X/\xi_X)\Lambda = -[\Phi_X]$, because

$$[X] = 1, \quad \frac{1}{2} ([X]^2 - [XX]) = 0, \quad \det X = 0. \quad (3.7)$$

Also, superpotential terms for the field E are given by

$$W_E = \varepsilon_{SB} (\mu_E [E\Theta_E] + \mu_E^2 [\Theta_E]). \quad (3.8)$$

Note that we cannot write the superpotential terms (3.6) and (3.8) without breaking the $U(1)_X$ symmetry explicitly, so that we have added a factor ε_{SB} in the superpotential terms (3.6) and (3.8).

In the superpotential terms (3.3), (3.6) and (3.8), we have assumed some specific forms of $U(1)_X$ symmetry breaking terms. As such symmetry breaking terms, in general, not only those given in (3.3), (3.6) and (3.8) but also many other terms are allowed. Therefore, at present, the forms of the ε_{SB} -terms are only ones which are required from a phenomenological point of view.

4 Concluding remarks

In conclusion, for the purpose of explaining the observed tribimaximal neutrino mixing, we have proposed a yukawaon model (1.4) [(3.4)] in the quark sector, where the $O(3)$ symmetry is broken into S_3 by the VEV $\langle \Phi_X \rangle_e$ on the e basis (the diagonal basis of $\langle Y_e \rangle$). The up-quark mass matrix given in Eq.(1.4) includes one parameter a_u . In the seesaw-type neutrino mass matrix $M_\nu = m_D M_R^{-1} m_D$, the Majorana mass matrix of the right-handed neutrinos $M_R \propto \langle Y_R \rangle$ is related to the up-quark mass matrix $M_u \propto \langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle$ as given in Eq.(2.5) which includes one parameter ξ . The parameters a_u and ξ can describe 5 observables (2 up-quark mass ratios and 3 neutrino oscillation parameters). The numerical results for the neutrino oscillation parameters excellently gives nearly tribimaximal mixing as shown in Table 1.

So far, we did not mention neutrino mass ratios, i.e. $R = \Delta m_{solar}^2 / \Delta m_{atm}^2$. The ratio R can favorably be fitted by adding [1] a term $\lambda'_R [Y_\nu Y_\nu \Theta_R]$ to the superpotential terms W_R (3.7) [(3.1)], or by adding [10] a term $(y'_R / \Lambda) \nu^c Y_\nu Y_\nu \nu^c$ to the would-be Yukawa interaction (1.7). Those terms play a role in shifting neutrino masses commonly without affecting neutrino mixing. However, the added parameter is exactly fixed by the observed value of R , so that there is no prediction in the neutrino sector.

Although the present model with (1.4) and (2.5) can describe values of 11 observables (4 quark mass ratios, 4 CKM mixing parameters, and 3 neutrino oscillation parameters) by adjusting 4 parameters (1 and 2 in the up- and down-quark sectors, respectively, and 1 in the Y_R sector), the fitting for down-quark masses and the CKM mixing parameters is poor as seen in Table 2, although the results are qualitatively not so bad. The two parameter description in the down-quark sector is too tight. We think that the present model is a step in the right direction to a unified yukawaon model. In the next step, we will investigate a further improvement of the down-quark sector.

In the present model, we have assumed an $O(3)$ flavor symmetry. The VEV relations which are derived from the superpotential under the $O(3)$ symmetry are valid only in specific flavor bases which are connected by orthogonal transformations. We have regarded the e -basis (the diagonal basis of $\langle Y_e \rangle$) as a specific basis, in which the relations from the $O(3)$ symmetry are valid and the VEV matrix $\langle \Phi_X \rangle$ takes a simple form (3.5).

The present approach based on a yukawaon model seems to provide a new view to a unified description of the masses and mixings differently from conventional mass matrix models. Under

few parameters in the quark sector, the model can predict not only all quark and lepton mass ratios but also the CKM matrix parameters and neutrino oscillation parameters, although an improvement is still needed as far as the down-quark sector is concerned. It is worthwhile taking the present model seriously as a promising model which can give a unified description of the quark and lepton masses and mixings. Further development of the model is expected.

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