

Yukawaons in the Quark Sectors

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Abstract

Based on the so-called “yukawaon” model, where the effective Yukawa coupling constants Y_f^{eff} ($f = e, \nu, u, d$) are given by vacuum expectation values (VEVs) of gauge singlet scalars (yukawaons) Y_f with 3×3 flavor components, i.e. $Y_f^{eff} = (y_f/\Lambda)\langle Y_f \rangle$, it is tried to describe VEV structures of the yukawaons Y_u and Y_d in the up- and down-quark sectors in terms of a VEV matrix $\langle \Phi_e \rangle$ of a ur-yukawaon Φ_e whose eigenvalues are given by $\langle \Phi_e \rangle \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$. Thereby, not only the CKM mixing parameters but also neutrino oscillation parameters are predicted under few adjustable parameters.

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1 Introduction

Recently, the author has proposed a new prescription for quark and lepton masses and mixings: We regard the Yukawa coupling constants in the standard model as “effective” coupling constants Y_f^{eff} ($f = e, \nu, u, d$) in an effective theory, and we consider that Y_f^{eff} originate in vacuum expectation values (VEVs) of new gauge singlet scalars Y_f , i.e.

$$Y_f^{eff} = \frac{y_f}{\Lambda} \langle Y_f \rangle, \quad (1.1)$$

where Λ is a scale of the effective theory. We refer the fields Y_f as “yukawaons” [1, 2] hereafter. Note that in the yukawaon model, Higgs scalars are the same as ones in the conventional model, i.e. we consider only two Higgs scalars H_u and H_d as an origin of the masses (not as an origin of the mass spectra). It should be noted that the yukawaons Y_f are gauge singlets.

The Froggatt-Nielsen model [3] is well known as a model which describes masses and mixings by a VEV value of a scalar ϕ : The hierarchical structure of the masses is explained by a multiplicative structure $(\langle \phi \rangle/\Lambda)^n$ under a U(1) flavor symmetry. In contrast to the Froggatt-Nielsen model, in the yukawaon model, the scalars Y_f have 3×3 flavor components, so that they are described, for example, as $\mathbf{3} \times \mathbf{3}^* = \mathbf{1} + \mathbf{8}$ of U(3) flavor symmetry or $(\mathbf{3} \times \mathbf{3})_S = \mathbf{1} + \mathbf{5}$ of O(3) flavor symmetry. The hierarchical structures of the quark and lepton masses are understood from hierarchical eigenvalues of $\langle Y_f \rangle$, not from the multiplicative structure $(\langle Y_f \rangle/\Lambda)^n$. Although our goal is to build a model which does not involve higher dimensional terms with $(1/\Lambda)^n$ ($n \geq 1$), we are obliged to accept would-be Yukawa interactions with a factor $1/\Lambda$ at present.

In the present paper, we assume an O(3) flavor symmetry, so that would-be Yukawa interactions are given by [4]

$$H_Y = \sum_{i,j} \frac{y_u}{\Lambda} u_i^c(Y_u)_{ij} q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} d_i^c(Y_d)_{ij} q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} \ell_i(Y_\nu)_{ij} \nu_j^c H_u + \sum_{i,j} \frac{y_e}{\Lambda} \ell_i(Y_e)_{ij} e_j^c H_d + h.c. + \sum_{i,j} y_R \nu_i^c(Y_R)_{ij} \nu_j^c, \quad (1.2)$$

where q and ℓ are $SU(2)_L$ doublet fields, and f^c ($f = u, d, e, \nu$) are $SU(2)_L$ singlet fields. All of the yukawaons Y_f ($f = u, d, \nu, e, R$) belong to $(\mathbf{3} \times \mathbf{3})_S = \mathbf{1} + \mathbf{5}$ of $O(3)$. In the definition (1.2) of Y_f , we can define diagonalizations of the VEV matrices $\langle Y_f \rangle$ ($f = u, d, e$) as

$$U_f^T \langle Y_f \rangle U_f \propto \text{diag}(m_{f1}, m_{f2}, m_{f3}), \quad (1.3)$$

and that of the seesaw-type neutrino mass matrix $M_\nu \propto \langle Y_\nu \rangle^T \langle Y_R \rangle^{-1} \langle Y_\nu \rangle$ as

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}), \quad (1.4)$$

so that we can express the quark mixing matrix [Cabibbo-Kobayashi-Maskawa matrix (CKM) matrix] V and lepton mixing matrix U as

$$V = U_u^\dagger U_d, \quad \text{and} \quad U = U_e^\dagger U_\nu, \quad (1.5)$$

respectively. In the interactions (1.2), in order to distinguish each Y_f from others, we have assumed a $U(1)_X$ symmetry in addition to the $O(3)$ flavor symmetry, and we have assigned $U(1)_X$ charges (“sector” charges, not “flavor” charges) as $Q_X(f^c) = -x_f$, $Q_X(Y_f) = +x_f$ and $Q_X(Y_R) = 2x_\nu$. The $SU(2)_L$ doublet fields q , ℓ , H_u and H_d are assigned as sector charges $Q_X = 0$.

In a supersymmetric (SUSY) yukawaon model, VEV structures of the yukawaons are obtained from SUSY vacuum conditions for a superpotential W . As a result, a VEV structure of $\langle Y_f \rangle$ in an f sector is described in terms of other yukawaon VEV matrices. In other words, a VEV structure of $\langle Y_f \rangle$ is given by observed mass matrices in other sectors. This is just a characteristic feature in the yukawaon approach. For example, in Ref.[4] (for improved versions, also see Refs.[1, 5]), a yukawaon model for the neutrino sector has been proposed: a neutrino mass matrix M_ν is given by

$$M_\nu \propto \langle Y_\nu \rangle \langle Y_R \rangle^{-1} \langle Y_\nu \rangle, \quad (1.6)$$

as well as a conventional seesaw mass matrix model, but the Dirac neutrino mass matrix m_ν^D (i.e. $\langle Y_\nu \rangle$) is given by $\langle Y_\nu \rangle \propto \langle Y_e \rangle$, so that a structure of the lepton mixing $U \neq \mathbf{1}$ originates in a structure of the Majorana mass matrix $\langle Y_R \rangle$ of the right-handed neutrinos ν^c . In (1.6), the Majorana mass matrix $\langle Y_R \rangle$ of the right-handed neutrinos has a curious form

$$\langle Y_R \rangle \propto \langle Y_e \rangle \langle \Phi_u \rangle + \langle \Phi_u \rangle \langle Y_e \rangle, \quad (1.7)$$

where $\langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle$, which is derived from a SUSY vacuum condition $\partial W / \partial \Theta_u = 0$ for superpotential terms

$$W_u = \mu_u [Y_u \Theta_u] + \lambda_u [\Phi_u \Phi_u \Theta_u]. \quad (1.8)$$

The relation (1.7) is derived from superpotential terms

$$W_R = \mu_R [Y_R \Theta_R] + \lambda_R [(Y_e \Phi_u + \Phi_u Y_e) \Theta_R]. \quad (1.9)$$

In order to obtain the lepton mixing matrix U , we evaluate the mass matrix (1.6) on the diagonal basis of the VEV matrix $\langle Y_e \rangle$ (we refer it as “ e -basis” and also we will refer a diagonal basis of $\langle Y_f \rangle$ as “ f -basis”):

$$\langle M_\nu \rangle_e \propto \langle Y_e \rangle_e \{ \langle Y_e \rangle_e \langle \Phi_u \rangle_e + \langle \Phi_u \rangle_e \langle Y_e \rangle_e \}^{-1} \langle Y_e \rangle_e, \quad (1.10)$$

where $\langle A \rangle_e$ denotes a form of the VEV matrix $\langle A \rangle$ on the e -basis. From a SUSY vacuum condition $\partial W / \partial \Theta_u = 0$, $\langle Y_u \rangle$ is given by

$$\langle Y_u \rangle = -\frac{\lambda_u}{\mu_u} \langle \Phi_u \rangle \langle \Phi_u \rangle, \quad (1.11)$$

so that a form of $\langle \Phi_u \rangle_u$ is given by

$$\langle \Phi_u \rangle_u \propto \text{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t}). \quad (1.12)$$

The form $\langle \Phi_u \rangle$ which we needs in the neutrino mass matrix (1.10) is $\langle \Phi_u \rangle_e$ in the e basis. However, we do not know the form although we know $\langle \Phi_u \rangle_d = V(\delta)^T \langle \Phi_u \rangle_u V(\delta)$, where $V(\delta)$ is the CKM matrix in the standard expression [6] and the observed value δ in the quark sector is $\delta \simeq 70^\circ$ [7]. Therefore, in the Ref.[4, 1, 5], we have put a phenomenological ansatz

$$\langle \Phi_u \rangle_e = V(\pi)^T \langle \Phi_u \rangle_u V(\pi), \quad (1.13)$$

on the analogy of $\langle \Phi_u \rangle_d = V(\delta)^T \langle \Phi_u \rangle_u V(\delta)$ and from a reason that $\langle \Phi_u \rangle_e$ must be real in the O(3) model. Usually, the so-called tribimaximal mixing [8] is understood based on discrete symmetries, while, in this model, the neutrino mass matrix (1.10) can give a nearly tribimaximal mixing $\sin^2 2\theta_{23} = 0.995$, $\tan^2 \theta_{12} = 0.553$ and $|U_{13}| = 0.001$ [4] by using observed values in quark sectors, without assuming any discrete symmetry. However, this successful result relies on the ad hoc assumption (1.13). We have no theoretical ground for the assumption (1.13).

Note that the matrix $V(\pi)$ corresponds to the matrix U_u^T which diagonalizes an up-quark mass matrix $M_u = (y_u/\Lambda)\langle Y_u \rangle_e \langle H_u \rangle$ in the e basis:

$$U_u^T \langle Y_u \rangle_e U_u = \langle Y_u \rangle_u \propto \text{diag}(m_u, m_c, m_t). \quad (1.14)$$

Therefore, if we give a quark mass matrix model at the e basis, we can obtain the neutrino mixing matrix U without relying such an ad hoc assumption. The purpose of the present paper is to investigate a yukawaon model in the quark sector which roughly gives reasonable lepton mixing as well as quark mixing. However, the purpose of the present paper is to give a main framework of the model which can give roughly reasonable masses and mixings for whole quark and lepton sectors, and it is not to give precise predictions numerically. We will search for a possible framework which can give rough description with few parameters for whole quark and lepton sectors. As we demonstrate in the next section, we will have 3 adjustable parameters (1 and 2 in the up- and down-quark sectors, respectively), and we try to describe 11 observables (4 quark mass ratios, 4 CKM mixing parameters, and 3 neutrino oscillation parameters). Especially, the one parameter in the up-quark sector will determine 5 observables (2 up-quark mass ratios and 3 neutrino oscillation parameters).

Our goal is to describe VEVs of all the yukawaons Y_f in terms of VEVs of only one yukawaon Φ_e . As a hint for such a unified yukawaon model, there is an interesting model, the so-called “democratic universal seesaw model” [9]: The quark mass matrices M_q ($q = u, d$) are given by $M_q = M_e^{1/2} M_Q^{-1} M_e^{1/2}$ (Q denotes a hypothetical heavy quark $Q = U, D$ correspondingly $q = u, d$) with a form $M_Q = (\text{democratic matrix}) + (\text{unit matrix})$. The scenario can be translated to a yukawaon model in the quark sectors: i.e.

$$\begin{aligned}\langle Y_u \rangle_e &\propto \langle \Phi_e \rangle_e (X + a_u e^{i\alpha_u} \mathbf{1}) \langle \Phi_e \rangle_e \\ \langle Y_d \rangle_e &\propto \langle \Phi_e \rangle_e (X + a_d e^{i\alpha_d} \mathbf{1}) \langle \Phi_e \rangle_e,\end{aligned}\tag{1.15}$$

where

$$X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.\tag{1.16}$$

Here, the VEV matrix $\langle \Phi_e \rangle$ is given by

$$\langle \Phi_e \rangle_e \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}),\tag{1.17}$$

which are obtained from superpotential terms

$$W_e = \mu_e [Y_e \Theta_e] + \lambda_e [\Phi_e \Phi_e \Theta_e],\tag{1.18}$$

similarly to Eq.(1.8). In fact, as we demonstrate in the next section, when we take suitable values of the parameters a_u and a_d , we can give reasonable predictions [9] not only for mass spectra of the up- and down-quarks but also for the CKM matrix. Regrettably, as see in the next section, a form U_u which is evaluated from (1.15) is in poor agreement with the form $V(\pi)^T$. The yukawaon model (1.10) in the neutrino sector can successfully lead to a nearly tribimaximal mixing by assuming the ad hoc relation (1.13) and by using input data from the quark sector, but without assuming any discrete symmetry. On the other hand, the yukawaon model (1.15) in quark sector can predict reasonable values not only mass spectra but also the CKM mixing under a suitable choice of the parameters (a_u, a_d) . Both models are equally attractive. Therefore, we want that both scenarios are compatible. In the present paper, we will propose a revised quark mass matrix model in which the VEV matrix $\langle Y_u \rangle$ is replaced with $\langle \Phi_u \rangle$. Then, the numerical results can give not only reasonable quark masses and mixings but also the observed fact $\sin^2 2\theta_{23} \simeq 1$ in the neutrino oscillations. Considering such the phenomenological success of the model, in Sec.3, a yukawaon model in the quark sector is discussed. In Sec.4, an alternative yukawaon model in the up-quark sector is proposed. Although the model can have a simpler structure of the superpotential and can improve the numerical predictions, we will be obliged to accept a higher dimensional term with $1/\Lambda^2$. Finally, Sec.5 is devoted to a summary and concluding remarks.

2 Phenomenological study

Table 1: An example of the ϕ dependence of the CKM mixing parameters. Input parameters are taken as $a_u = 0.010$ and $(a_d, \alpha_d) = (-2/3, -8.5^\circ)$.

| ϕ | $ V_{us} $ | $ V_{cb} $ | $ V_{ub} $ | $ V_{td} $ |
|-------------|------------|------------|------------|------------|
| 0° | 0.2197 | 0.4440 | 0.0142 | 0.1000 |
| 90° | 0.2219 | 0.3554 | 0.0119 | 0.0812 |
| 180° | 0.2204 | 0.0586 | 0.0030 | 0.0148 |
| 270° | 0.2183 | 0.2725 | 0.0083 | 0.0602 |

In the present section, we give numerical studies based on phenomenological mass matrices (1.10), (1.15) and so on. Since the aim of the present paper is not to obtain the best fit values of the parameters in the model, and it is to see a main framework of the model qualitatively, an energy scale used in the present evaluations is not required to be so rigorous. Exactly speaking, VEV relations in the present model are valid at $\mu = \Lambda \sim 10^{15}$ GeV, which comes from a neutrino mass $m_\nu \sim \langle H_u^0 \rangle^2 / \Lambda$ in the model (1.10). (Here, we have assumed that all VEVs of the yukawaons and coefficients with a dimension of mass in the superpotential are of the order of Λ .) Since the O(3) flavor symmetry is completely broken at $\mu = \Lambda$, the effective coupling constants $Y_f^{eff} = (y_f / \Lambda) \langle Y_f \rangle$ evolve as in the standard model below the scale Λ . We know that the quark mass ratios are not so sensitive to the energy scale [11]. As seen in (2.3) and (2.6) below, the observed quark mass values [11] still have large errors. Since our concern is in the quark mixing matrix V and lepton mixing matrix U which are defined by (1.5), for simplicity, we evaluate the mass matrices (1.15) at the energy scale $\mu = m_Z$, so that numerical results in the present section should not be taken too strictly. In the present model, quark mass matrices will be described in terms of the charged lepton masses, so that for numerical estimates, we will use values of the running masses $m_e(\mu)$, $m_\mu(\mu)$ and $m_\tau(\mu)$, which do not satisfy the well-known charged lepton mass relation [10]. Although the yukawaon model [2] was first proposed with the aim of understanding such the charged lepton mass relation, in the present paper, we do not adopt such a yukawaon model in the charged lepton sector. We will use the observed values [11] of the running charged lepton masses as input values.

The model (1.15) can give reasonable quark mass spectra by taking $a_u \simeq \pm 0.01$ and $a_d \simeq -0.6$ for the up- and down-quark sectors, respectively. Although the straightforward application $V = U_u^\dagger U_d$ cannot give the observed CKM mixing parameters, if we phenomenologically introduce a phase matrix

$$P = \text{diag}(1, 1, e^{i\phi}), \quad (2.1)$$

and we replace the expression of the CKM matrix $V = U_u^\dagger U_d$ with $V = U_u^\dagger P U_d$, then we can obtain reasonable values of the CKM mixing parameters by taking $\phi = \pi$ as shown in Table 1 (and also Table 2). However, the matrix U_u which has been obtained from the diagonalization of the up-quark mass matrix (1.15) cannot give reasonable values of neutrino mixing parameters, especially, of $\sin^2 2\theta_{23}$, as seen in Table 3.

Therefore, in the present paper, we pay attention to the observed fact $\sqrt{m_{ui}/m_{uj}} \sim m_{di}/m_{dj}$, and we consider that a VEV matrix which corresponds to $\langle Y_d \rangle$ is not $\langle Y_u \rangle$, but $\langle \Phi_u \rangle$.

Table 2: CKM mixing parameters versus a_u (a case of $P = \text{diag}(1, 1, -1)$)

| a_u | m_{u1}/m_{u2} | m_{u2}/m_{u3} | $ m_{d1}/m_{d2} $ | $ m_{d2}/m_{d3} $ | $ V_{us} $ | $ V_{cb} $ | $ V_{ub} $ | $ V_{td} $ |
|--------|-----------------|-----------------|-------------------|-------------------------|------------|------------|------------|------------|
| | | | $a_d = -2/3$ | $\alpha_d = -8.5^\circ$ | | | | |
| -0.011 | 0.00373 | -0.00362 | 0.0557 | 0.0316 | 0.2205 | 0.0616 | 0.0033 | 0.0155 |
| -0.010 | 0.00373 | -0.00328 | 0.0557 | 0.0316 | 0.2205 | 0.0614 | 0.0033 | 0.0155 |
| +0.010 | 0.00378 | +0.00303 | 0.0557 | 0.0316 | 0.2204 | 0.0586 | 0.0030 | 0.0148 |
| +0.011 | 0.00378 | +0.00332 | 0.0557 | 0.0316 | 0.2204 | 0.0586 | 0.0030 | 0.0148 |
| | | | $a_d = -2/3$ | $\alpha_d = -6.0^\circ$ | | | | |
| -0.011 | 0.00373 | -0.00328 | 0.0980 | 0.0238 | 0.2923 | 0.0439 | 0.0028 | 0.0146 |
| -0.010 | 0.00373 | -0.00328 | 0.0980 | 0.0238 | 0.2923 | 0.0439 | 0.0028 | 0.0146 |
| +0.010 | 0.00378 | +0.00303 | 0.0980 | 0.0238 | 0.2923 | 0.0419 | 0.0024 | 0.0140 |
| +0.011 | 0.00378 | +0.00332 | 0.0980 | 0.0238 | 0.2923 | 0.0419 | 0.0024 | 0.0140 |

 Table 3: Neutrino mixing parameters at $|a_u| \sim 0.01$.

| a_u | $\sin^2 2\theta_{23}$ | $\tan^2 \theta_{12}$ | $ U_{13} $ |
|--------|-----------------------|----------------------|------------|
| -0.011 | 0.2373 | 1.0335 | 0.0608 |
| -0.010 | 0.2345 | 1.0334 | 0.0610 |
| +0.010 | 0.3298 | 0.6675 | 0.0631 |
| +0.011 | 0.3354 | 0.6637 | 0.0632 |

We change the model (1.15) into the following model:

$$\begin{aligned}
 \langle \Phi_u^0 \rangle_e &\propto \langle \Phi_e \rangle_e (X + a_u \mathbf{1}) \langle \Phi_e \rangle_e, \\
 \langle Y_d \rangle_e &\propto \langle \Phi_e \rangle_e (X + a_d e^{i\alpha_d} \mathbf{1}) \langle \Phi_e \rangle_e.
 \end{aligned}
 \tag{2.2}$$

Here, we have denoted Φ_u defined in Eq.(2.2) as Φ_u^0 . (The reason will be understood later.) Since we have assumed that all relations from the $O(3)$ symmetry are also valid in the u basis, we have taken the parameter $a_u e^{i\alpha_u}$ real.

First, we search for a value of the parameter a_u which can give reasonable predicted values for the up-quark mass ratios [11]

$$\frac{m_u}{m_c} = \frac{0.0022_{-0.0008}^{+0.0013}}{(0.0021_{-0.0009}^{+0.0012})}, \quad \frac{m_c}{m_t} = \frac{0.0036_{-0.0005}^{+0.0006}}{(0.0026_{-0.0006}^{+0.0007})}, \tag{2.3}$$

at $\mu = m_Z$ (at $\mu = \Lambda_{GUT} = 2 \times 10^{16}$ GeV with $\tan \beta = 10$), i.e. $\sqrt{m_u/m_c} = 0.045_{-0.010}^{+0.013}$ and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$. We find that the value $a_u \simeq -0.56$ is in favor of the observed up-quark mass ratios as seen in Table 4.

Next, we calculate $\langle \Phi_u \rangle_e$ in order to evaluate neutrino oscillation parameters. If we regard $\langle \Phi_u \rangle_e$ in Eq.(1.10) as $\langle \Phi_u^0 \rangle_e$ given in Eq.(2.2), we cannot obtain favorable values of the neutrino mixing parameters for any values of a_u . Note that, for the parameter value $a_u \sim -0.6$, the ratios

Table 4: Neutrino mixing parameters at $a_u \sim -0.6$.

| a_u | $\sqrt{m_{u1}/m_{u2}}$ | $\sqrt{m_{u2}/m_{u3}}$ | $\sin^2 2\theta_{23}$ | $\tan^2 \theta_{12}$ | $ U_{13} $ |
|-------|------------------------|------------------------|-----------------------|----------------------|------------|
| -0.54 | 0.0300 | 0.0747 | 0.945 | 0.742 | 0.0099 |
| -0.56 | 0.0425 | 0.0571 | 0.985 | 0.703 | 0.0128 |
| -0.58 | 0.0630 | 0.0427 | 0.999 | 0.655 | 0.0166 |
| -0.60 | 0.1000 | 0.0310 | 0.981 | 0.595 | 0.0223 |
| -0.62 | 0.1741 | 0.0216 | 0.894 | 0.520 | 0.0323 |

v_{u1}^0/v_{u3}^0 and v_{u2}^0/v_{u3}^0 of the VEV eigenvalues $v_{ui}^0 \equiv \langle \Phi_u^0 \rangle$ are negative. We define Φ_u so that the eigenvalues of $\langle \Phi_u \rangle$ are positive. That is, in this case, since $v_{u2}^0/v_{u3}^0 < 0$ and $v_{u1}^0/v_{u3}^0 < 0$ at $a_u \sim -0.6$, we define

$$\langle \Phi_u \rangle_u = P_u \langle \Phi_u^0 \rangle_u P_u, \quad (2.4)$$

where

$$P_u = \text{diag}(i, i, 1). \quad (2.5)$$

[In the $O(3)$ model, the form of $\langle \Phi_u \rangle$ must be symmetric, so that we cannot consider a form $\langle \Phi_u \rangle_u = P_u \langle \Phi_u^0 \rangle_u$ with $P_u = \text{diag}(-1, -1, 1)$.] As seen in Table 4, however, the predicted values of $\tan^2 \theta_{12}$ are not so in good agreement with the observed value [13] $\tan^2 \theta_{12} = 0.469_{-0.041}^{+0.047}$, although the predicted values of $\sin^2 2\theta_{23}$ are in favor of the observed data [12] $\sin^2 2\theta_{23} = 1.00_{-0.13}$. (If we use $\langle \Phi_u^0 \rangle_e$ defined in (2.2) for $\langle \Phi_u \rangle_e$ in Eq.(1.10), we will obtain more unwelcome results $\sin^2 2\theta_{23} = 0.207$ and $\tan^2 \theta_{12} = 1.032$ irrelevantly of the value of a_u .) If we take $a_u \simeq -0.62$, we can obtain reasonable values of the neutrino oscillation parameters, but we cannot get reasonable up-quark mass ratios. We think that some fine tuning may be required for components related to the first family. Anyhow, for the moment, we are satisfied with the result $\sin^2 2\theta_{23} \simeq 1$, and next, let us search for a reasonable parameter (a_d, α_d) which gives the down-quark mass ratios [11]

$$\frac{m_d}{m_s} = \frac{0.053_{-0.029}^{+0.051}}{(0.054_{-0.030}^{+0.058})}, \quad \frac{m_s}{m_b} = \frac{0.019 \pm 0.006}{(0.017_{-0.005}^{+0.006})}, \quad (2.6)$$

at $\mu = m_Z$ (at $\mu = \Lambda_{GUT} = 2 \times 10^{16}$ GeV with $\tan \beta = 10$) and the observed CKM mixing parameters.

In Table 5, we demonstrate predicted values of the CKM mixing parameters versus (a_d, α_d) . Here, we have taken $a_u = -0.58$ as a typical value of a_u . In order to obtain reasonable values of the CKM mixing parameters, in the original democratic model [9] (1.15), we have been obliged to take a phenomenological phase matrix $P = \text{diag}(1, 1, -1)$ given in Eq.(2.1). In contrast to the old model (1.15), in the present model, we do not need such a unwelcome phase matrix. As seen in Table 5, the case with $a_u = -0.58$ and $(a_d, \alpha_d) \simeq (-0.63, 2^\circ)$ can give reasonable values of $|V_{12}|$ and $|V_{23}|$, although the predicted values of $|V_{13}|$ and $|V_{31}|$ are somewhat larger than the observed values. Considering that our prediction of $\tan^2 \theta_{12}$ is also somewhat larger than the observed value, the present model will need a further improvement, as far as the first generation

Table 5: CKM mixing parameters versus (a_d, α_d) . The value of a_u is fixed at $a_u = -0.58$.

| a_d | α_d | $ V_{us} $ | $ V_{cb} $ | $ V_{ub} $ | $ V_{td} $ |
|-------|------------|------------|------------|------------|------------|
| -0.62 | 2° | 0.1698 | 0.0384 | 0.0075 | 0.0121 |
| -0.63 | 2° | 0.2272 | 0.0448 | 0.0088 | 0.0163 |
| -0.62 | 3° | 0.1868 | 0.0438 | 0.0084 | 0.0128 |
| -0.63 | 3° | 0.2386 | 0.0493 | 0.0096 | 0.0163 |
| -0.62 | 5° | 0.2089 | 0.0574 | 0.0109 | 0.0140 |
| -0.63 | 5° | 0.2456 | 0.0613 | 0.0117 | 0.0164 |

is concerned. However, from the qualitative point of view, it worthwhile noticing that the model can roughly predict not only the quark mass ratios but also the CKM mixing parameters and neutrino oscillation parameters by using only the three input parameters a_u and (a_d, α_d) .

3 Yukawaons in the quark sector

In the previous section, for the up-quark sector, we have assumed two ur-yukawaons Φ_u^0 and Φ_u , which are related to Y_u as $\langle Y_u \rangle_u \propto \langle \Phi_u \rangle_u \langle \Phi_u \rangle_u \propto \langle \Phi_u^0 \rangle_u \langle \Phi_u^0 \rangle_u$. Since the eigenvalues of $\langle \Phi_u^0 \rangle$ and $\langle \Phi_u \rangle$ are different from each other in their phases, we cannot identify $\langle \Phi_u^0 \rangle$ as $\langle \Phi_u \rangle$. We consider that the field Φ_u is a true one, and $\langle \Phi_u^0 \rangle_u$ is only a provisional one which is, for convenience, introduced in order to describe $\langle \Phi_u \rangle$ via Eq.(2.4). Therefore, we consider the following superpotential in the quark sector:

$$\begin{aligned}
 W_q = & \mu_u [Y_u \Theta_u] + \lambda_u [\Phi_u \Phi_u \Theta_u] + \frac{\xi'_u}{\Lambda} [Q_u \Phi_u Q_u \Theta'_u] + \frac{\xi_u}{\Lambda} [\Phi_e S_u \Phi_e \Theta'_u] \\
 & + \mu_d [Y_d \Theta_d] + \frac{\xi_d}{\Lambda} [\Phi_e S_d \Phi_e \Theta_d],
 \end{aligned} \tag{3.1}$$

so that, from $\partial W / \partial \Theta'_u = 0$, we obtain

$$\langle \Phi_u \rangle_e = -\frac{\xi_u}{\xi'_u} \langle Q_u \rangle_e^{-1} \langle \Phi_e \rangle_e \langle S_u \rangle_e \langle \Phi_e \rangle_e \langle Q_u \rangle_e^{-1}. \tag{3.2}$$

Here, the field Q_u has been introduced in order to make the eigenvalues of $\langle \Phi_u \rangle$ positive. For the moment, we assume

$$\langle Q_u \rangle_u \propto P_u \equiv \text{diag}(i, i, 1), \tag{3.3}$$

so that a form $\langle Q_u \rangle_e$ in the e basis is given by

$$\langle Q_u \rangle_e \propto R_u P_u R_u^T, \tag{3.4}$$

where R_u is defined by $R_u^T \langle \Phi_u^0 \rangle_e R_u = \langle \Phi_u^0 \rangle_u$. For the VEV matrix form of $\langle S_q \rangle_e$, for the moment, we also assume $\langle S_q \rangle_e$ has a form

$$\langle S_q \rangle_e = k_q (X + a_q e^{i\alpha_q} \mathbf{1}) \quad (q = u, d). \tag{3.5}$$

The VEV matrix form (3.5) breaks the $O(3)$ symmetry into S_3 on the e basis.

In the superpotential (3.1), $U(1)_X$ charges of the fields S_u and S_d have assigned as $Q_X(S_u) = \frac{1}{2}x_u - x_e + 2Q_X(Q_u)$ and $Q_X(S_d) = x_d - x_e$, respectively. Therefore, we can consider a model with $Q_X(S_u) = Q_X(S_d)$ unless $Q_X(Q_u) = 0$. [If $Q_X(Q_u) = 0$, the result $Q_X(S_u) = Q_X(S_d)$ leads to $x_d = \frac{1}{2}x_u$, so that Φ_u and Y_d have the same charge, and a mixing between Φ_u and Y_d will be caused.] In the model with $Q_X(S_u) = Q_X(S_d)$, instead of the fields S_u and S_d , we can consider a model with alternative two fields Φ_X and E which are defined as

$$S_q = \Phi_X + a_q e^{i\alpha_q} E \quad (q = u, d), \quad (3.6)$$

And whose VEV matrices are given by

$$\langle \Phi_X \rangle_e \propto X, \quad \langle E \rangle_e \propto \mathbf{1}. \quad (3.7)$$

Superpotential terms for the field Φ_X are Given by

$$W_X = \varepsilon \left\{ \frac{\xi_X}{\Lambda} [\Phi_X \Phi_X \Phi_X \Theta_X] + \lambda_X [\Phi_X \Phi_X \Theta_X] \right\}, \quad (3.8)$$

with $(\lambda_X/\xi_X)\Lambda = [\Phi]$, because eigenvalues of a 3×3 Hermitian matrix A is completely determined by the equation

$$AAA - [A]AA + c_1 A + c_0 \mathbf{1} = 0, \quad (3.9)$$

where

$$c_1 = \frac{1}{2} ([A]^2 - [AA]), \quad c_0 = -\det A. \quad (3.10)$$

On the other hand, superpotential terms for the field E are given by

$$W_E = \varepsilon (\mu_E [E \Theta_E] + \mu_E^2 [\Theta_E]). \quad (3.11)$$

Note that we cannot write the superpotential terms (3.8) and (3.11) without breaking the $U(1)_X$ symmetry explicitly. We assume that the $U(1)_X$ symmetry can explicitly be broken with an order of the small value ε .

4 Alternative model

By the way, in the model (3.1) [i.e. (2.2)], we have consider a $\Phi_u \leftrightarrow Y_d$ correspondence. However, there is somewhat a feeling of wrongness in the correspondence. We feel that a Y_u - Y_d correspondence is natural rather than the Φ_u - Y_d correspondence. Therefore, in this section, we consider an alternative model, where Φ_u in the previous section is replaced with Y_u . Then, the would-be Yukawa interaction term in the up-quark sector must be replaced as

$$\frac{y_u}{\Lambda} u^c Y_u q H_u \rightarrow \frac{y_u}{\Lambda^2} u^c Y_u Y_u q H_u, \quad (4.1)$$

by taking $Q_X = -2x_u$ instead of $Q_X = -x_u$. The superpotential terms (3.1) in the quark sector is replaced with

$$W_q = \frac{\xi'_u}{\Lambda} [Q_u Y_u Q_u \Theta_u] + \frac{\xi_u}{\Lambda} [\Phi_e S_u \Phi_e \Theta_u] + \mu_d [Y_d \Theta_d] + \frac{\xi_d}{\Lambda} [\Phi_e S_d \Phi_e \Theta_d], \quad (4.2)$$

where S_q ($q = u, d$) are defined by (3.6). Also, the relation in the neutrino sector (1.9) is rewritten as

$$W_R = \mu_R[Y_R\Theta_R] + \lambda_R[(Y_e Y_u + Y_u Y_e)\Theta_R], \quad (4.3)$$

where $\langle Y_u \rangle_e$ is given by

$$\langle Y_u \rangle_e = U_u^* \langle Y_u \rangle_u U_u^\dagger, \quad (4.4)$$

with positive eigenvalues of $\langle Y_u \rangle_u$.

Note that we do not need the superpotential terms (1.8), because $\langle Y_u \rangle \langle Y_u \rangle$ in (4.2) can automatically become diagonal when $\langle Y_u \rangle$ is diagonal. In the previous model (3.1), since we need the VEV relation (1.11) on the u basis, we have taken the VEV matrix $\langle \Phi_u \rangle_e$ real. (In the O(3) model, VEV relations from SUSY vacuum conditions are valid on the bases which are connected to each other by orthogonal transformations. Since we have assumed that our VEV relations are valid on the e basis, the u basis must be connected to the e basis by an orthogonal transformation if we want that the relations are also valid on the u basis.) Since the revised model (4.1) does not need such terms (1.8), we can take $\langle Y_u \rangle_e$ complex, i.e. we can take $\alpha_u \neq 0$ in Eq.(3.6). Therefore, in the present model, the numerical predictions given in Tables 4 and 5 can considerably be improved by taking 4 parameter fitting.

Thus, although the present model has a natural and simple structure, we obliged to accept a higher dimensional term with $1/\Lambda^2$ as seen in (4.1). Our goal is to build a yukawaon model without any higher dimensional term. Therefore, the present model is a model which goes toward the opposite.

5 Concluding remarks

In conclusion, we have proposed a yukawaon model in the quark sector, where the O(3) symmetry is broken into S_3 by the VEV $\langle \Phi_X \rangle_e$ on the e basis (the diagonal basis of $\langle Y_e \rangle$). Roughly speaking, the model can give reasonable values of 11 observables (4 quark mass ratios, 4 CKM mixing parameters, and 3 neutrino oscillation parameters) by adjusting 3 parameters (1 and 2 in the up- and down-quark sectors, respectively). Especially, the parameter a_u in the up-quark sector can determine 5 observables (2 up-quark mass ratios and 3 neutrino oscillation parameters). The numerical results for the neutrino oscillation parameters are roughly in favor of the observed ones although the predicted value $\tan^2 \theta_{12} \simeq -0.65$ is somewhat larger than the observed value [13] $\tan^2 \theta_{12} = 0.469_{-0.041}^{+0.049}$ (although the original model [4] with the ad hoc ansatz (1.13) has given an excellent nearly tribimaximal mixing). Although some of predicted values are still over the experimental error values, we think that the present model is a step in the right direction to a unified yukawaon model. In the next step, we will take some small additional terms into consideration. In the present model, we could not give superpotential terms which definitely leads to the VEV form of $\langle Q_u \rangle$ given in Eq.(3.3). This is also a future task to us.

In the conventional approach in a quark and lepton mass matrix model, the mass matrices M_f are, in principle, independent of each other, although we put constraints by assuming flavor symmetries. In the yukawaon approach, the mass matrices (VEV matrices of the yukawaons) are directly connected to each other. In the present model, we have assumed an O(3) flavor symmetry. The VEV relations which are derived from the superpotential under the O(3) symmetry are valid only in specific flavor bases which are connected by orthogonal transformations. We

have regarded the e -basis (the diagonal basis of $\langle Y_e \rangle$) as a specific basis, in which the relations from the $O(3)$ symmetry are valid and the VEV matrices $\langle S_q \rangle$ take a simple form (3.4).

The present approach based on a yukawaon model seems to provide a new view to a unified description of the masses and mixings differently from conventional mass matrix models. The model can predict, under few parameters in the quark sector, not only all quark and lepton mass ratios but also the CKM matrix parameters and neutrino oscillation parameters, although an improvement is still needed as far as the first family is concerned. It is worthwhile taking the present model seriously as a promising model which can give a unified description of the quark and lepton masses and mixings. Further development of the model is expected.

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