

How Can CP Violation in the Neutrino Sector be Large in a $2 \leftrightarrow 3$ Symmetric Model?

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Abstract

Based on a neutrino mass matrix model in which a 2-3 symmetry is only broken by a phase parameter, it is investigated how the lepton mixing matrix can deviate from the so-called tribimaximal mixing under a condition that CP is maximally violated.

1 Introduction

Recent data in neutrino oscillation experiments [1, 2] have showed that the lepton mixing matrix U is fairly in favor of the so-called tribimaximal mixing [3] U_{TB} . If U is exactly $U = U_{TB}$, then CP violation in the lepton sector is absolutely forbidden. Our interest is in how the CP violation in the lepton sector can be large and how U can deviate from the tribimaximal mixing U_{TB} .

Meanwhile, we know that a neutrino mass matrix M_ν with a $2 \leftrightarrow 3$ symmetry [4] can lead to a maximal ν_2 - ν_3 mixing. Of course, a rigorous requirement of the $2 \leftrightarrow 3$ symmetry for the neutrino and charged lepton sectors cannot give a realistic lepton mixing matrix U because of $SU(2)_L$ symmetry [5]. We have to bring a breaking term into the $2 \leftrightarrow 3$ symmetric mass matrix by hand. A quark and lepton mass matrix model [6] has been proposed, in which the 2-3 symmetry is broken only by a phase parameter: A mass matrix M_f is given by a form

$$M_f = P(\delta_f) \widehat{M}_f P^\dagger(\delta_f), \quad (1.1)$$

where

$$P(\delta) = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}), \quad (1.2)$$

$$\widehat{M}_f = \begin{pmatrix} 0 & a_f & a_f \\ a_f & b_f & -c_f \\ a_f & -c_f & b_f \end{pmatrix}, \quad (1.3)$$

where a_f , b_f and c_f are real parameters. [We have defined the expression (1.3) such as $m_{f3} > m_{f2} > -m_{f1} > 0$ for $b_f > c_f > 0$. See Eq.(A.4) in Appendix.] Here, the matrix \widehat{M}_f is exactly symmetric under the $2 \leftrightarrow 3$ flavor exchange, but the 2-3 symmetry for the mass matrix M_f is

broken by the phase factor $\delta_{f3} - \delta_{f2} \neq 0$. A short review of the diagonalization of the Hermitian mass matrix (1.1) is given in Appendix.

In this paper, we investigate CP violation in the lepton sector. Various models in which the 2-3 symmetry is broken only by phase parameters can be considered. The most naive way is to consider a model $M_\nu = P(\delta_\nu)\widehat{M}_\nu P(\delta_\nu)$ similar to the form (1.1), and such a model has been discussed in Ref.[6]. However, in the present paper, we take another idea that the CP violation in the lepton sector is caused by phase parameters which cannot be removed by a form $M_\nu = P(\delta_\nu)\widehat{M}_\nu P(\delta_\nu)$ differently from those in the quark sector. The simplest realization of such a model is to assume a neutrino mass matrix form

$$M_\nu = \begin{pmatrix} 0 & a_\nu & a_\nu \\ a_\nu & b_\nu e^{i(\phi+\chi)} & -c_\nu \\ a_\nu & -c_\nu & b_\nu e^{i(\phi-\chi)} \end{pmatrix}, \quad (1.4)$$

where a_ν , b_ν and c_ν are real parameters. Here, the 2-3 symmetry is broken only by the phase parameter χ as

$$\frac{M_{\nu 22} - M_{\nu 33}}{M_{\nu 22} + M_{\nu 33}} = i \tan \chi, \quad (1.5)$$

while CP is broken by the phase parameter ϕ . We consider that the existence of these phase parameters is characteristic only in a Majorana mass matrix and we cannot consider such the phases in Dirac mass matrices (we assume that those are Hermitian). The mass matrix can lead to a nearly tribimaximal mixing when CP is maximally broken in the lepton sector. [For the charged lepton sector, we assume the matrix form (1.3).]

It is hard to solve the mass matrix model (1.4) analytically. Therefore, in Sec.2, we investigate the model (1.4) numerically. The model (1.4) has four parameters $a \equiv a_\nu/b_\nu$, $c \equiv c_\nu/b_\nu$, ϕ and χ for four observable quantities $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $|U_{13}|^2$ and $R \equiv \Delta m_{sol}^2/\Delta m_{atm}^2$. We will find that if we put a maximal CP violation hypothesis, we can obtain interesting predictions. Finally, Sec.3 is devoted to summary and concluding remarks.

2 Lepton mixing matrix

The lepton mixing matrix U is given by

$$U = U_e^\dagger U_\nu, \quad (2.1)$$

where U_e and U_ν are defined by

$$U_e^\dagger M_e U_e = \text{diag}(m_e, m_\mu, m_\tau), \quad (2.2)$$

$$U_\nu^\dagger M_\nu U_\nu^* = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}). \quad (2.3)$$

A form of U_e can readily be given by the analytic form $U_e = P(\delta_e)R_e$ given in Eq.(A.1). In contrast to the case of U_e , it is hard to express U_ν with an analytic form. Therefore, in this paper, we discuss the parameter dependences of the lepton mixing matrix U by numerical studies.

Since, for U_e , we fix the mixing angle θ_e by inputting the charged lepton masses at $\mu = m_Z$, the remaining free parameters for U are a_ν/b_ν , c_ν/b_ν , ϕ and χ . (Since the value of θ_e is very small, i.e. $\theta_e = 3.936^\circ$, the essential rotation in the lepton mixing matrix U comes from the neutrino sector U_ν .) As stated in the previous section, the phase parameter χ breaks the $2 \leftrightarrow 3$ symmetry, and ϕ causes the CP violation. In contrast to the phase parameters ϕ and χ in the lepton sector, there are two phase parameters α and β in the quark sector, which are defined by Eq.(A.8). The parameter β is only related to the breaking of the $2 \leftrightarrow 3$ symmetry and α is only related to the CP violation as shown in Eq.(A.9). However, note that those parameters in the quark sector do not affect quark mass values, while the phase parameters ϕ and χ affect not only the neutrino mixing matrix U_ν but also the neutrino mass values. In this paper, we will fix the phase parameters ϕ and χ by requiring a maximal CP violation in the lepton sector. The remaining parameters a_ν/b_ν and c_ν/b_ν can, in principle, be fixed by the observed values of

$$R = \frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} = \frac{m_{\nu 2}^2 - m_{\nu 1}^2}{m_{\nu 3}^2 - m_{\nu 2}^2}, \quad (2.4)$$

(we define as $m_{\nu 2}^2 > m_{\nu 1}^2$) and $\tan^2 \theta_{sol} = |U_{12}|^2/|U_{11}|^2$. Then, we can predict values of the rephasing invariant [7] $J = \text{Im}(U_{11}U_{22}U_{12}^*U_{21}^*)$, and the neutrino oscillation parameters $\sin^2 2\theta_{atm} = 4|U_{23}|^2|U_{33}|^2$, and $|U_{13}|^2$. However, the observed values of R and $\tan^2 \theta_{sol}$ are not so accurate, at present, to determine the parameter values $a = a_\nu/b_\nu$ and $c = c_\nu/b_\nu$. Therefore, in this paper, we will show the predictions of $|J|$, $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $|U_{13}|^2$ and R as a function of the parameters a and c under the requirement of the maximal CP violation.

We consider that the phenomenological mass matrix form (1.4) is given at an energy scale $\mu = M_Z$ (M_Z is the Z -boson mass). Since the present model gives the neutrino mass hierarchy $m_{\nu 1}^2 \ll m_{\nu 2}^2 \ll m_{\nu 3}^2$ (not $m_{\nu 1}^2 \simeq m_{\nu 2}^2 \ll m_{\nu 3}^2$), we can regard that the renormalization group equation effects are not so large for the mass ratios and mixing in the present model (see, for example, Ref.[8]). Therefore, the numerical calculation will be done at $\mu = M_Z$, though we suppose that a breaking of the 2-3 symmetry is caused at a TeV energy scale.

Note that in the following subsections A-C, we sometimes show figures with specific input parameter values. However, those input values are temporary ones for convenience to demonstrate parameter dependence of the model, and the values do not mean final best-fit values in the present analysis. The final values are obtained from iteration of the analysis A \rightarrow B \rightarrow C.

A. Behavior of J versus phase parameters

Our interest is in the maximal CP violation. The maximal CP violation is defined as follow: the rephasing invariant J takes its maximal value for the CP violating mass matrix parameters. The CP violating mass matrix parameter is a phase parameter α defined by Eq.(A.8) in the quark sector [the model (1.1)], while it corresponds to the phase parameter ϕ in the present model (1.4). We illustrate ϕ -dependence of J for typical values of χ in Figs.1. Since $\phi \rightarrow -\phi$ (and $\chi \rightarrow -\chi$) means $U_\nu \rightarrow U_\nu^*$, so that it means $U \rightarrow U^*$ because U_e is real. Note that cases with (a, c) and $(-a, -c)$ give identical results for the observable quantities, while those give different results for cases with $(a, -c)$ and $(-a, c)$. As far as the shape of $|J|$ versus ϕ is concerned, the cases with $ac < 0$ and $ac > 0$ are similar, but the maximal values of $|J|$ are

different from each other as seen in Figs.1 (a) and (b). Hereafter, we will refer the cases with $ac < 0$ and $ac > 0$ to as Cases A and B, respectively. As seen in Fig.1, the maximal $|J|$ takes place at $\phi \simeq \pm 13^\circ$ for the cases $\chi = 90^\circ$ and $\chi = 120^\circ$ (and $\phi \simeq \pm 12^\circ$ for the case $\chi = 60^\circ$) for both cases A and B. (Thus, the maximum point is slightly dependent on the parameter χ .) The case $\phi \simeq 13^\circ$ ($\phi \simeq 12^\circ$) gives a maximal J with $J > 0$, while $\phi \simeq -13^\circ$ ($\phi \simeq -12^\circ$) gives a maximal $|J|$ with $J < 0$. In this section, we take a standpoint that the maximal CP violation means a maximal J ($J > 0$). Hereafter, we confine ourselves to investigating a parameter region $0 \leq \phi \leq \pi/2$. Especially, since our interest is in the maximal CP violation, we will investigate parameter dependences for the case $\chi = \pi/2$ in which the 2-3 symmetry is maximally broken. On the other hand, in the limit of $\phi \rightarrow 0$, a case $\chi \rightarrow \pi/2$ (and also $\chi \rightarrow -\pi/2$) corresponds to a maximal 2-3 symmetry violation as seen in Eq.(1.5). Therefore, we confine ourselves to investigating a parameter region $0 \leq \chi \leq \pi/2$.

B. (a, c) dependence

Next, we investigate (a, c) -dependence of the observable values for $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $|U_{13}|^2$, and R in order to give more realistic predictions of them. Since our concern is to predict the observable quantities at a point in which CP is maximally violated, we illustrate the (a, c) dependence at $\chi = \pi/2$ and $\phi = 13^\circ$. (The value of $\phi = 13^\circ$ is not a final result, because the value of ϕ which gives a maximal $|J|$ in Fig.1 is dependent on the values of a and c .)

Correspondingly to Cases A and B, a -dependences of the predicted values are illustrated in Figs.2 (a) and (b), respectively, where the parameter c is fixed at a typical value $c = 0.95$ in both figures (a) and (b). We find that the present observed data [1, 2]

$$\tan^2 \theta_{sol} = 0.469_{-0.041}^{+0.047} \quad (\theta_{sol} = 34.4_{-1.2}^{+1.3} \text{ degrees}) \quad (2.5)$$

require $a = -(0.26_{-0.03}^{+0.02})$ and $a = +(0.29_{-0.03}^{+0.04})$ for Cases A and B, respectively, and

$$R_{obs} = \frac{(7.59 \pm 0.21) \times 10^{-5} \text{ eV}^2}{(2.19_{-0.13}^{+0.14}) \times 10^{-3} \text{ eV}^2} = (3.47_{-0.26}^{+0.43}) \times 10^{-2} \quad (2.6)$$

require $a = -(0.26 \pm 0.03)$ and $a = +(0.29_{-0.03}^{+0.04})$ for Cases A and B, respectively. From overall view, in Case A a value $a = -0.255$ can give favorable predictions of $\tan^2 \theta_{atm}$ and R , so that we will use the the value $a = -0.255$ for Case A hereafter. For Case B, we will use a value $a = +0.270$ hereafter, although the value does not so excellent agreement with the observed values (2.5) and (2.6) as compared with Case A.

Also, c -dependence is illustrated in Fig.3 (a) and (b), which correspond to Cases A and B, respectively. Here, the parameter a is fixed at $a = -0.255$ for Case A (and $a = +0.270$ for Case B) from the results in Fig.2. We find that the predicted values of $\tan^2 \theta_{sol}$ and R are not so sensitive to the value of c . If we use a parameter value $c = 0.95$ for both Cases A and B, which is favorable to the data, we obtain predictions $(\tan^2 \theta_{sol}, R) = (0.468, 0.0347)$ and $(0.443, 0.0350)$ correspondingly to the input values $(a = -0.255, c = 0.95)$ and $(a = +0.270, c = 0.95)$.

C. Phase parameter dependence

Again, in Fig.4, we illustrate behaviors of the predicted values J versus the phase parameter ϕ , together with the behaviors of $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $|U_{13}|^2$ and R under the maximal 2-3 symmetry violation $\chi = \pi/2$. Here, the parameter values $(a, c) = (-0.255, 0.95)$ and $(a, c) = (+0.270, 0.95)$ are taken in the figures (a) and (b), respectively, from the study in the previous subsection.

From Fig.4, we find that the values of $\sin^2 2\theta_{atm}$ and $|U_{13}|^2$ are insensitive to the value of ϕ , while the predicted values of $\tan^2 \theta_{sol}$ and R are highly sensitive to the value of ϕ . Exactly speaking, in Fig.4, since the parameters a and c have been taken so that the values $\tan^2 \theta_{sol}$ and R are reasonably fitted, the values of $\tan^2 \theta_{sol}$ and R are not predictions in Fig.4. Our predictions are for $\sin^2 2\theta_{atm}$ and $|U_{13}|^2$: $\sin^2 2\theta_{atm} \simeq 0.98$ and $|U_{13}|^2 \simeq 0.007$ for Case A, and $\sin^2 2\theta_{atm} \simeq 0.93$ and $|U_{13}|^2 \simeq 0.035$ for Case B.

D. Predictions under the maximal CP violation hypothesis

It is interesting to assume a hypothesis of the maximal CP violation. As seen in Fig.4, although the value of J_{max} , $J_{max} \simeq 0.039$, in Case A is larger than $J_{max} \simeq 0.018$ in Case B, it does not mean that Case A is ruled out under this hypothesis. The maximal J is required only for the phase parameters, not for the mass matrix parameters a_ν , b_ν and c_ν . Therefore, both Cases A and B are allowed under the maximal CP violation hypothesis. If we require a larger value of J , we have to choose Case B. However, the Case B predicts $\sin^2 2\theta_{atm} \leq 0.93$, which contradicts the present atmospheric neutrino data [9] $\sin^2 2\theta_{atm} \geq 0.96$. Therefore, we conclude that Case B should be ruled out, so that the mass matrix parameters a_ν and c_ν must have the opposite sign to each other. In contrast to this conclusion, note that a sign of the parameter a_f in a model (1.1) can be taken freely as seen in Eqs. (A.1) - (A.4).

In Fig.5, we illustrate the more detailed behavior of the physical observable quantities under the maximal CP violation hypothesis. Here, although Figs.5 (a) and (b) are similar to Fig.2 (a) and Fig.3 (b), respectively, Figs.5 (a) and (b) are illustrated under the maximal CP violation hypothesis, so that the value ϕ is always taken as it gives the maximal J for each parameter set of a and c . From Fig.5, we conclude that $\sin^2 2\theta_{atm} \simeq 0.98$ and $|U_{13}|^2 \simeq 0.007$ for the present data (2.5) and (2.6). In Fig.6, we also illustrate the contour plots for $\sin^2 2\theta_{atm}$ (Fig.(a)), $|U_{13}|^2 \times 10$ (Fig.(b)), and J (Fig.(c)) in addition to $R \times 10$ and $\tan^2 \theta_{sol}$ given in Eqs. (2.5) and (2.6) in the (a, c) parameter plane under the maximal CP violation hypothesis for each parameter set of a and c .

4 Summary

In conclusion, we have assumed a neutrino mass matrix in which a 2-3 flavor symmetry is violated only by a phase parameter, and which can lead to a nearly tribimaximal neutrino mixing, and thereby, we have investigated behaviors of the neutrino oscillation parameters under the maximal CP violation hypothesis. We find that the predicted values of $\sin^2 2\theta_{atm}$ and $|U_{13}|^2$ are insensitive to the input values R and $\tan^2 \theta_{sol}$ for $R \simeq 0.035$ and $\tan^2 \theta_{sol} \simeq 0.5$:

$$\sin^2 2\theta_{atm} \simeq 0.98, \quad |U_{13}|^2 \simeq 0.01. \quad (4.1)$$

Especially, we are interested in the case with $|R| = 0.0321$ which is a lower bound of the observed value of $|R|$ as shown in Eq.(2.6), because the value gives us an upper bound of the predicted

value for $\sin^2 2\theta_{atm}$ and a lower bound of the predicted value for $|U_{13}|^2$. As seen in Fig.6, our model rules out the following values

$$\sin^2 2\theta_{atm} > 0.989, \quad |U_{13}|^2 < 0.0046, \quad (4.2)$$

if neutrino oscillation data establish $|R| > 0.321$. On the other hand, if the present upper bound of $|R|$, $|R| = 0.0390$, is established, the following regions

$$\sin^2 2\theta_{atm} < 0.978, \quad |U_{13}|^2 > 0.0098, \quad (4.3)$$

are also ruled out. Those bounds (4.2) and (4.3) are within reach of near future experiments. Therefore, if future experiments give us values in the regions (4.2) and (4.3), the present model will be ruled out. On the contrary, if future experiments report observed values within

$$\begin{aligned} 0.989 \geq \sin^2 2\theta_{atm} \geq 0.978, \quad 0.0046 \leq |U_{13}|^2 \leq 0.0098, \\ 0.021 \geq J \geq 0.013, \end{aligned} \quad (4.4)$$

correspondingly to $0.0321 \leq |R| \leq 0.0390$ and $0.428 \leq \tan^2 \theta_{sol} \leq 0.516$, the neutrino mass matrix form (1.4) will become a promising candidate of the neutrino mass matrix form.

Also, note that the predicted value of $\tan^2 \theta_{sol}$ is highly correlated to the predicted value of R under the values (4.1). This result is shown in Fig.7 by using only the observable quantities with fixing typical values for $\sin^2 2\theta_{atm}$ such as $\sin^2 2\theta_{atm} = 0.980$ and 0.985 . This figure is also useful as a touchstone of the present model.

The neutrino mass matrix (1.4) is a very simple form and it has phase parameters which are characteristic only in a Majorana mass matrix. We expect that the results will be checked in the very near future experiments.

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Appendix

We give a short review of the diagonalization of the mass matrix (1.1) and the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix as an example of the flavor mixing matrix. (Although we have denoted as $(M_f)_{11} = 0$ for simplicity, it is not essential for the following discussions. For the case $(M_f)_{11} \neq 0$ in Eq.(1.1), it is only necessary that the parameter values b_f and $\delta_{f3} - \delta_{f2}$ are redefined under a common shift of the mass eigenvalues $m_{f_i} \rightarrow m_{f_i} + (\widehat{M}_f)_{11}$.)

The matrix \widehat{M}_f given in Eq.(1.3) is diagonalized by a rotation

$$R_f = R_1\left(-\frac{\pi}{4}\right)R_3(\theta_f) = \begin{pmatrix} \cos \theta_f & \sin \theta_f & 0 \\ -\frac{1}{\sqrt{2}} \sin \theta_f & \frac{1}{\sqrt{2}} \cos \theta_f & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \sin \theta_f & \frac{1}{\sqrt{2}} \cos \theta_f & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (A.1)$$

as

$$R_f^T \widehat{M}_f R_f = D_f \equiv \text{diag}(m_{f1}, m_{f2}, m_{f3}), \quad (A.2)$$

where

$$\cos \theta_f = \sqrt{\frac{m_{f2}}{m_{f2} - m_{f1}}}, \quad \sin \theta_f = \sqrt{\frac{-m_{f1}}{m_{f2} - m_{f1}}}, \quad (A.3)$$

$$\begin{aligned} m_{f1} &= \frac{1}{2} \left(b_f - c_f - \sqrt{8a_f^2 + (b_f - c_f)^2} \right), \\ m_{f2} &= \frac{1}{2} \left(b_f - c_f + \sqrt{8a_f^2 + (b_f - c_f)^2} \right), \\ m_{f3} &= b_f + c_f. \end{aligned} \quad (A.4)$$

Here and hereafter, we use the following notations for the rotation matrices

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_3(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (A.5)$$

where $c = \cos \theta$ and $s = \sin \theta$. When we apply this model to quark mass matrices, we obtain the CKM quark mixing matrix V as follows:

$$V = U_u^\dagger U_d = R_u^T P(\delta) R_d = R_3^T(\theta_u) R_1^T(-\frac{1}{4}\pi) P(\delta) R_1(-\frac{1}{4}\pi) R_3(\theta_d), \quad (A.6)$$

where $\delta_i = \delta_{ui} - \delta_{di}$ ($i = 1, 2, 3$) and $\tan \theta_f = \sqrt{-m_{f1}/m_{f2}}$. We can rewrite the factor $R_1^T(-\frac{1}{4}\pi) P(\delta) R_1(-\frac{1}{4}\pi)$ as

$$\begin{aligned} R_1^T(-\frac{1}{4}\pi) P(\delta) R_1(-\frac{1}{4}\pi) &= \begin{pmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & \frac{1}{2}(e^{i\delta_2} + e^{i\delta_3}) & \frac{1}{2}(-e^{i\delta_2} + e^{i\delta_3}) \\ 0 & \frac{1}{2}(-e^{i\delta_2} + e^{i\delta_3}) & \frac{1}{2}(e^{i\delta_2} + e^{i\delta_3}) \end{pmatrix} \\ &= e^{i(\alpha+\delta_1)} \begin{pmatrix} e^{-i\alpha} & 0 & 0 \\ 0 & \cos \beta & i \sin \beta \\ 0 & i \sin \beta & \cos \beta \end{pmatrix} = e^{i(\alpha+\delta_1)} P_1(-\alpha) P_3(-\frac{1}{2}\pi) R_1(\beta) P_3(\frac{1}{2}\pi), \end{aligned} \quad (A.7)$$

where

$$\alpha = \frac{1}{2}(\delta_3 + \delta_2) - \delta_1, \quad \beta = \frac{1}{2}(\delta_3 - \delta_2), \quad (A.8)$$

$P_1(\delta) = \text{diag}(e^{i\delta}, 1, 1)$ and $P_3(\delta) = \text{diag}(1, 1, e^{i\delta})$. Since P_3 and R_3 are commutable each other, the phase matrices $P_3(-\pi/2)$ and $P_3(\pi/2)$ can be eliminated by redefining phases of the up- and down-quarks, respectively, so that we can obtain a CKM matrix form as follows:

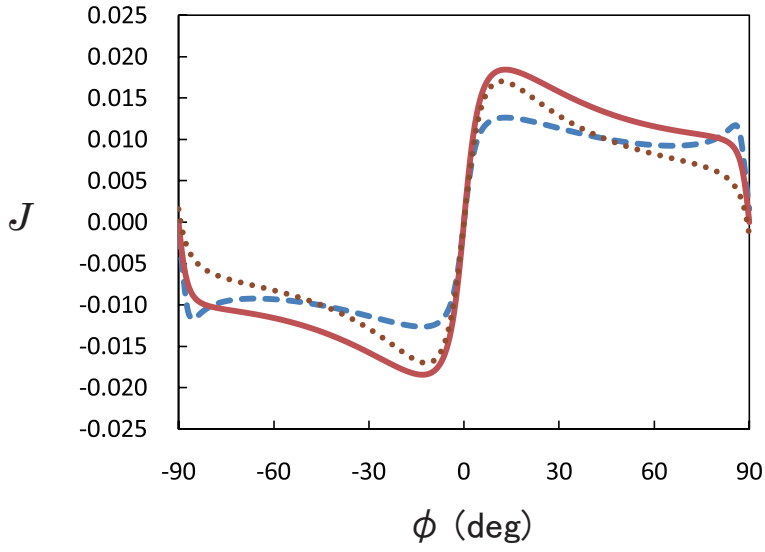
$$V = R_3^T(\theta_u) P_1(-\alpha) R_1(\beta) R_3(\theta_d). \quad (A.9)$$

This expression (A.9) is well-known as the Fritzsch-Xing expression [10] of the CKM matrix. Note that the phase parameter α still plays a role as a CP violating phase, while the parameter

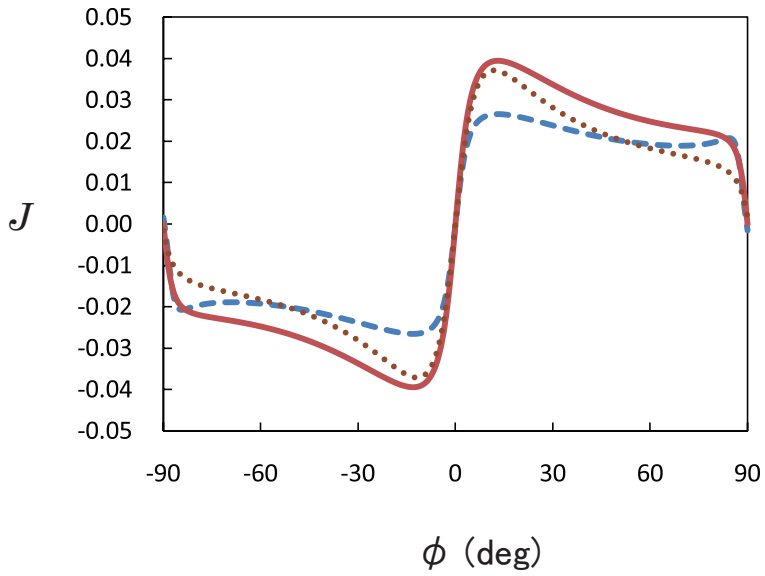
β plays only a role as a rotation parameter θ_{23} . It is known that the model is in favor of the observed values of the CKM parameters under a hypothesis that the CP violation is maximally violated, i.e. $\alpha = \pi/2$.

References

- [1] D. G. Michael *et al.*, MINOS collaboration, Phys. Rev. Lett. **97**, 191801 (2006); J. Hosaka, *et al.*, Super-Kamiokande collaboration, Phys. Rev. **D74** 032002 (2006).
- [2] B. Aharmim, *et al.*, SNO collaboration, Phys. Rev. Lett. **101**, 111301 (2008). Also, see S. Abe, *et al.*, KamLAND collaboration, Phys. Rev. Lett. **100**, 221803 (2008).
- [3] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B458**, 79 (1999); Phys. Lett. **B530** (2002) 167; Z.-z. Xing, Phys. Lett. **B533**, 85 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. **B535**, 163 (2002); Phys. Lett. **B557**, 76 (2003); E. Ma, Phys. Rev. Lett. **90**, 221802 (2003); C. I. Low and R. R. Volkas, Phys. Rev. **D68**, 033007 (2003).
- [4] T. Fukuyama and H. Nishiura, hep-ph/9702253, in Proceedings of *the International Workshop on Masses and Mixings of Quarks and Leptons*, Shizuoka, Japan, 1997, edited by Y. Koide (World Scientific, Singapore, 1998), p. 252; R. N. Mohapatra and S. Nussinov, Phys. Rev. **D60**, 013002, (1999); E. Ma and M. Raidal, Phys. Rev. Lett. **87**, 011802 (2001); C. S. Lam, Phys. Lett. **B507**, 214 (2001); K. R. S. Balaji, W. Grimus and T. Schwetz, Phys. Lett. **B508**, 301 (2001); W. Grimus and L. Lavoura, Acta Phys. Pol. **B32**, 3719 (2001). For a recent work, for example, see, R. N. Mohapatra, JHEP **0410**, 027 (2004); W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka and M. Tanimoto, Nucl. Phys. **B713**, 151 (2005).
- [5] Y. Koide and E. Takasugi, Phys. Rev. **D77**, 016006 (2008).
- [6] Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi and T. Fukuyama, Phys. Rev. **D66**, 093006 (2002).
- [7] C. Jarlskog, Phys. Rev. Lett. **55**, 1839 (1985); O. W. Greenberg, Phys. Rev. **D32**, 1841 (1985); I. Dunietz, O. W. Greenberg and D.-d. Wu, Phys. Rev. Lett. **55**, 2935 (1985); C. Hamzaoui and A. Barroso, Phys. Rev. **D33**, 860 (1986).
- [8] K. Matsuda and H. Nishiura, Phys. Rev. **D69**, 117302 (2004).
- [9] Y. Suzuki, an invited talk at Lepton-Photon 2009, Hamburg, Germany, 17-22 August, 2009.
- [10] H. Fritzsch and Z.-z. Xing, Phys. Lett. **B413**, 396 (1997).

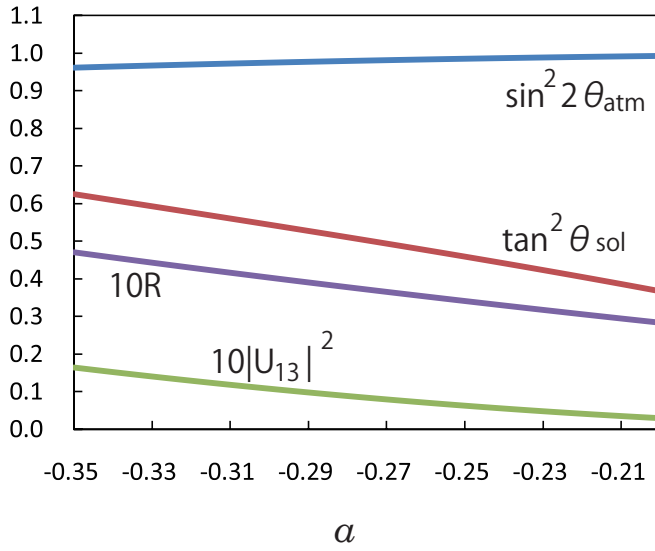


(a)

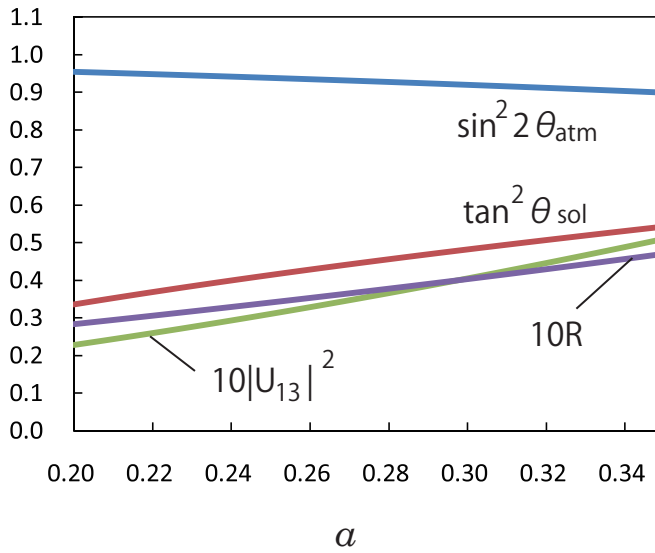


(b)

Fig. 1 Behavior of the rephasing invariant J versus the CP violation parameter ϕ . Figures (a) and (b) are illustrated for typical values $(a, c) = (-0.27, +0.95)$ and $(a, c) = (+0.27, +0.95)$, respectively. The dashed, solid, and dotted curves correspond to cases with $\chi = 120^\circ$, $\chi = 90^\circ$, and $\chi = 60^\circ$, respectively.

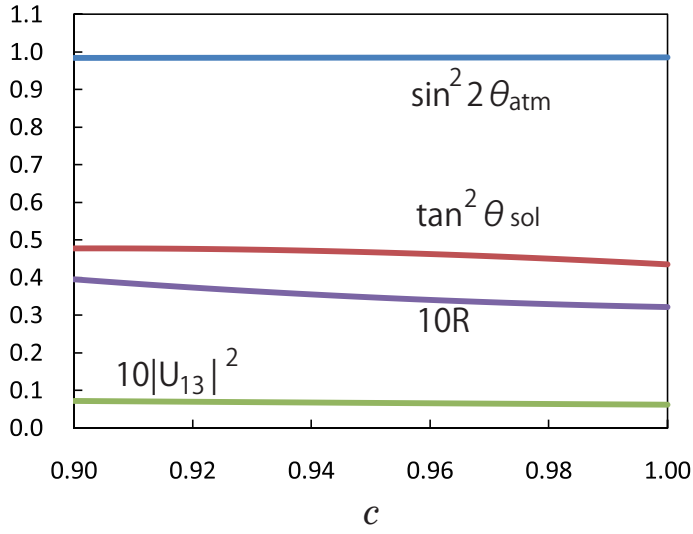


(a)

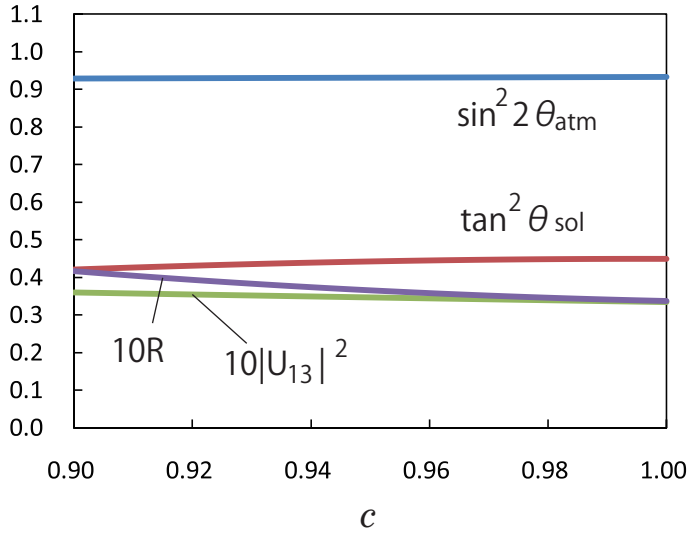


(b)

Fig. 2 a -dependence of the predicted values for $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $R \times 10$ and $|U_{13}|^2 \times 10$. Other parameters are fixed at typical values $c = c_\nu/b_\nu = 0.95$, $\chi = \pi/2$, and $\phi = 13^\circ$. Figures (a) and (b) correspond to Case A with $ac < 0$ and Case B with $ac > 0$, respectively.

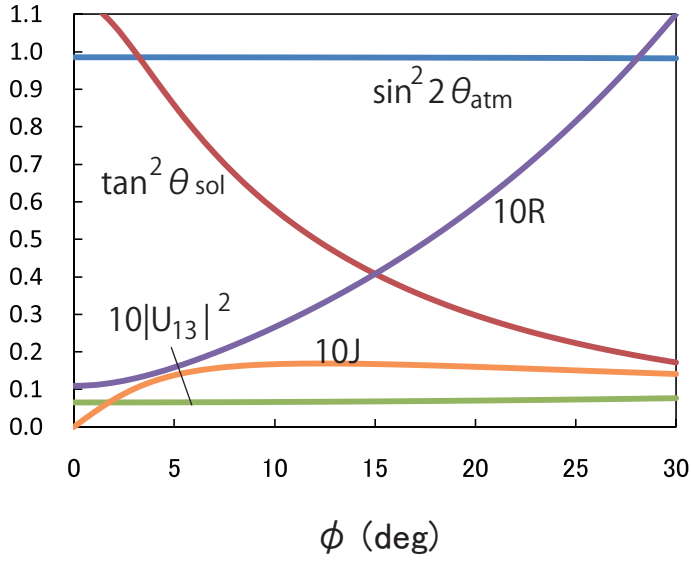


(a)

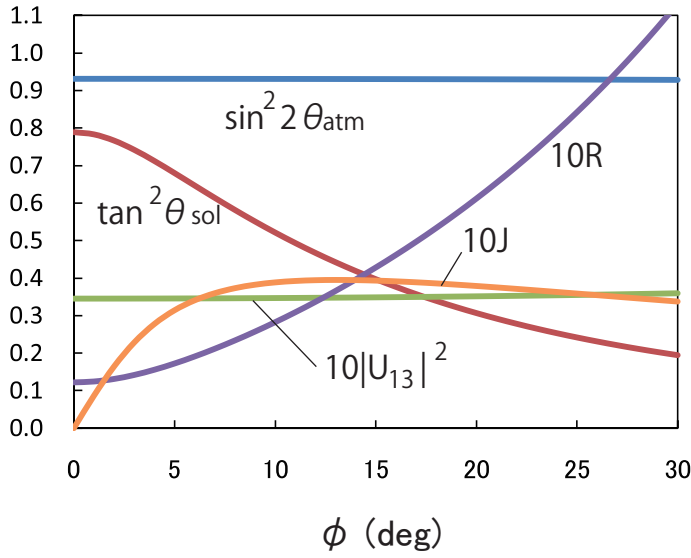


(b)

Fig. 3 c -dependence of the predicted values for $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $R \times 10$, and $|U_{13}|^2 \times 10$. Figures (a) and (b) correspond to Case A with $ac > 0$ and Case B with $ac < 0$, respectively and those are illustrated by taking input values $a = -0.255$ and $a = +0.27$, respectively. Other parameters are fixed at typical values $\chi = \pi/2$ and $\phi = 13^\circ$.

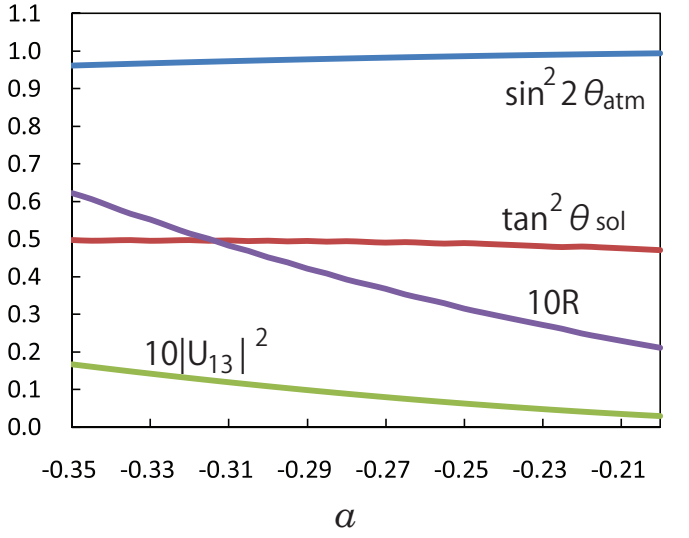


(a)

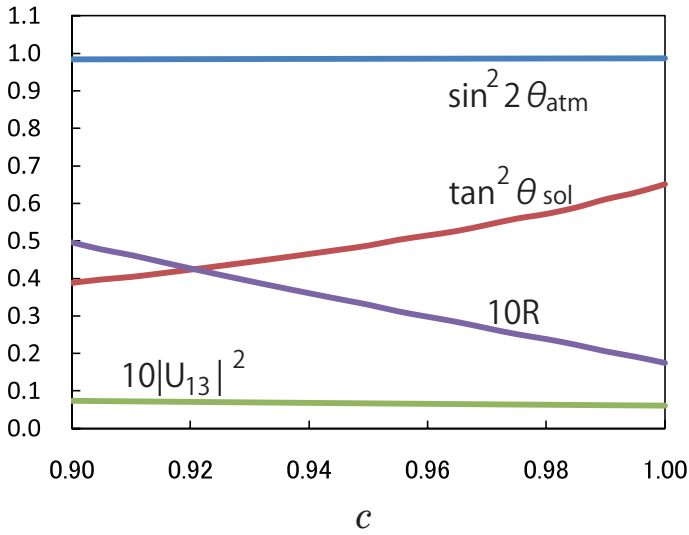


(b)

Fig. 4 Behavior of J , $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $R \times 10$, and $|U_{13}|^2 \times 10$, versus the CP violating phase parameter ϕ under the maximal 2-3 symmetry violation $\chi = \pi/2$. Figures (a) and (b) correspond to Case A with $ac < 0$ and Case B with $ac > 0$, respectively, and those are illustrated by taking input values $(a, c) = (-0.255, 0.95)$ and $(a, c) = (+0.27, 0.95)$, respectively.



(a)



(b)

Fig. 5 a -dependence (Fig. (a)) and c -dependence (Fig. (b)) of the predicted values for J , $\sin^2 2\theta_{atm}$, $\tan^2 \theta_{sol}$, $R \times 10$, and $|U_{13}|^2 \times 10$ under the maximal CP violating hypothesis: the value of ϕ is taken as J is maximal for each parameter set of a and c . Here, the phase parameter χ is fixed at $\chi = 90^\circ$ at which J takes the maximal value, and the parameter c is taken as $c = 0.95$ for Fig. (a) and a as $a = -0.255$ for Fig. (b).

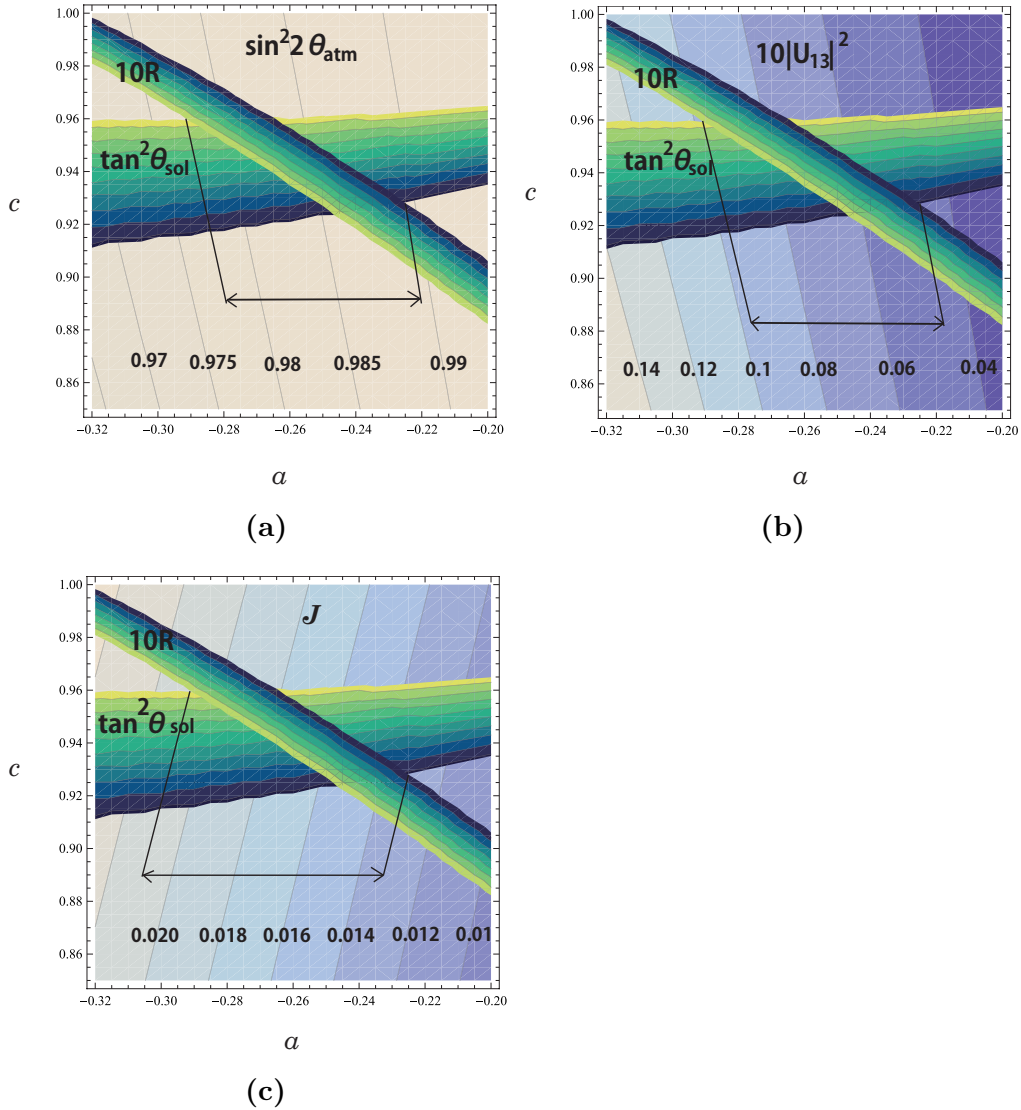


Fig. 6 Contour plots of predicted values of $\sin^2 2\theta_{atm}$ (Fig. (a)) , $|U_{13}|^2 \times 10$ (Fig. (b)), and J (Fig. (c)) in the a - c parameter plane under the maximal CP violating hypothesis: the value of ϕ is taken as J is maximal for each parameter set of a and c . Here, the phase parameter χ is fixed at $\chi = 90^\circ$ at which J takes the maximal value. We also present contour plots of $R \times 10$ and $\tan^2 \theta_{sol}$ which are correspond to the experimental constraints, $0.0321 \leq |R| \leq 0.0390$ and $0.428 \leq \tan^2 \theta_{sol} \leq 0.516$, respectively. Here the darker region corresponds to smaller contour values. Note that the overlapped region of the contours of $10R$ and $\tan^2 \theta_{sol}$ is the allowed region of the a - c parameter plane of the model.

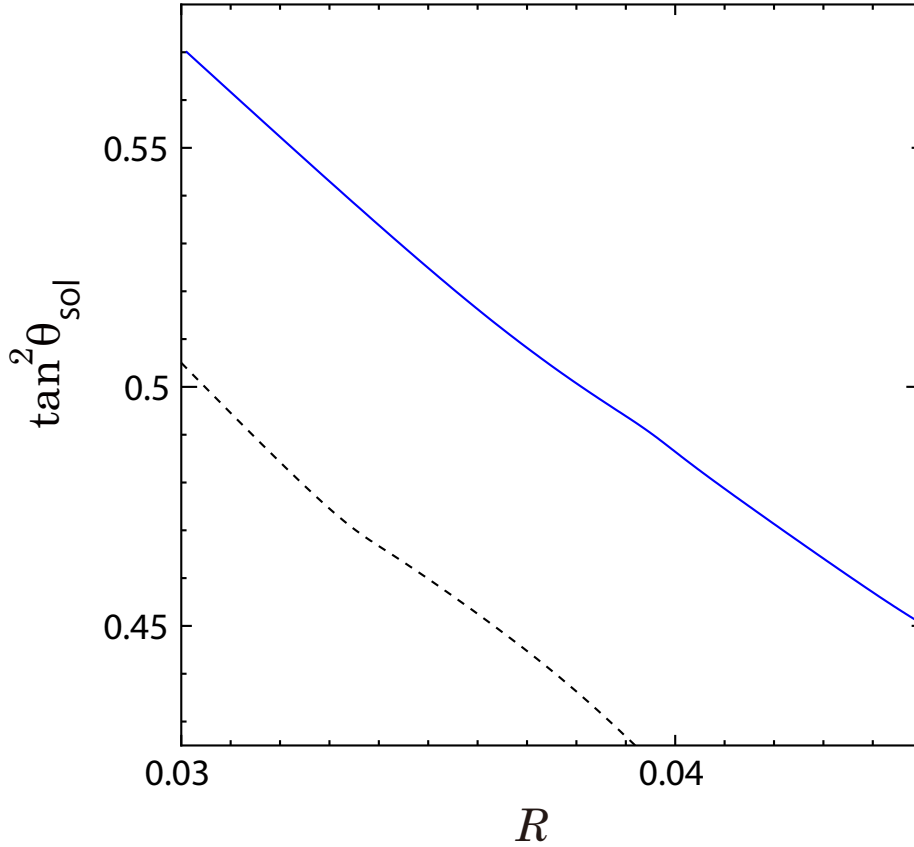


Fig. 7 Behavior of $\tan^2 \theta_{sol}$ vs R under the maximal CP violation hypothesis with taking typical values of $\sin^2 2\theta_{atm}$. The solid and dashed curves correspond to the cases with $\sin^2 2\theta_{atm} = 0.980$ and $\sin^2 2\theta_{atm} = 0.985$, respectively. The predicted values of $|U_{13}|^2$ for $\sin^2 2\theta_{atm} = 0.980$ and $\sin^2 2\theta_{atm} = 0.985$ are $|U_{13}|^2 \simeq 0.0087$ and $|U_{13}|^2 \simeq 0.0066$, respectively, in this parameter region of (a, c) .