U(3)-Flavor Nonet Scalar as an Origin of the Flavor Mass Spectra

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Abstract

According to an idea that the quark and lepton mass spectra originate in a VEV structure of a U(3)-flavor nonet scalar Φ, the mass spectra of the down-quarks and charged leptons are investigated. The U(3) flavor symmetry is spontaneously and completely broken by non-zero and non-degenerated VEVs of Φ, without passing any subgroup of U(3). The ratios $(m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$ and $\sqrt{m_e m_\mu m_\tau}/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3$ are investigated based on a toy model.

1 Introduction

The observed mass spectra of the quarks and leptons might provide a promising clue for a unified understanding of the quarks and leptons. In investigating an origin of the flavor mass spectra, we may expect that an approach based on symmetries will be promising. However, when we want to introduce a flavor symmetry into our mass matrix model, we always encounter an obstacle, i.e. a no-go theorem [1] in flavor symmetries. The theorem asserts that we cannot bring any flavor symmetry into a mass matrix model as far as we consider a mass generation mechanism based on the standard model. The premises to derive the theorem are as follow: (i) the $SU(2)_L$ symmetry is unbroken; (ii) there is only one Higgs scalar in each sector (e.g. H_u and H_d for up- and down-quark sectors, respectively); (iii) 3 eigenvalues of Y_f in each sector are non-zero and no-degenerate. Therefore, we have three options [2] to evade this no-go theorem: (A) a model with more than two Higgs scalars in each sector; (B) a model with an explicit flavor symmetry breaking term; (C) a model with a new scalar whose vacuum expectation values (VEVs) yield effective Yukawa coupling constants. The approach (B) has adopted by many authors in phenomenological studies of flavor symmetries. However, we want a model without such an explicit symmetry breaking term. The approach (A) induces a flavor-changing neutral current (FCNC) problem [3]. In order to suppress the FCNC effects, we must make those Higgs scalars heavy except for one of the linear combinations of those scalars. However, it is not so easy to build such a reasonable suppression mechanism without an explicit symmetry breaking term.

Therefore, in the present paper, we take a great interest in the option (C). For example, we consider the following superpotential terms:

$$W_Y = \sum_{i,j} \frac{y_u}{M}(Y_u)_{ij} Q_i H_u U_j + \sum_{i,j} \frac{y_d}{M}(Y_d)_{ij} Q_i H_d D_j$$

$$+ \sum_{i,j} \frac{y_\nu}{M}(Y_\nu)_{ij} L_i H_u N_j + \sum_{i,j} \frac{y_e}{M}(Y_e)_{ij} L_i H_d E_j + h.c. + y_R \sum_{i,j} N_i (M_R)_{ij} N_j, \tag{1.1}$$

where Y_f ($f = u, d, \nu, e$) are not coupling constants, but U(3)-flavor nonet fields [4, 5, 6], and Q and L are quark and lepton $SU(2)_L$ doublet fields, respectively, and U, D, N, and E are $SU(2)_L$ singlet matter fields. The mass parameter M denotes an energy scale of the effective theory. If we assume the following terms for an additional U(3)-nonet (gauge singlet) scalar Φ in the superpotential

$$W_\Phi = \lambda_\phi \mathrm{Tr}[\Phi\Phi\Phi] + m_{\phi\phi}\mathrm{Tr}[\Phi\Phi] + \mu_\phi^2 \mathrm{Tr}[\Phi] + \lambda_e \mathrm{Tr}[\Phi\Phi Y_e] + m_{ee}\mathrm{Tr}[Y_e Y_e], \tag{1.2}$$

we can obtain relations

$$3\lambda_\phi \Phi\Phi + 2m_{\phi\phi}\Phi + \mu_\phi^2 \mathbf{1} + \lambda_e(\Phi Y_e + Y_e \Phi) = 0, \tag{1.3}$$

$$\lambda_e \Phi\Phi + 2m_{ee} Y_e = 0, \tag{1.4}$$

from SUSY vacuum conditions $\partial W/\partial\Phi = 0$ and $\partial W/\partial Y_e = 0$, respectively. (Here, for simplicity, we have drop the contribution form W_Y.) Therefore, we can obtain a bilinear mass relation for the charged leptons

$$\langle Y_e \rangle = -\frac{\lambda_e}{2m_{ee}} \langle\Phi\rangle\langle\Phi\rangle, \tag{1.5}$$

from Eq.(1.4). This is entirely favorable for charged lepton mass relation as we state later. By substituting Eq.(1.5) into Eq.(1.3), we obtain

$$c_3 \Phi\Phi\Phi + c_2 \Phi\Phi + c_1 \Phi + c_0 \mathbf{1} = 0, \tag{1.6}$$

where

$$c_3 = \frac{\lambda_e^2}{m_{ee}}, \quad c_2 = -3\lambda_\phi, \quad c_1 = -2m_{\phi\phi}, \quad c_0 = -\mu_\phi^2. \tag{1.7}$$

Thus, if we give values of the coefficients c_n ($n = 3, 2, 1, 0$), we can completely determine three eigenvalues of $\langle\Phi\rangle$, so that we can give a charged lepton mass spectrum from Eq.(1.5). Especially, it is worthwhile noticing that a relation between $\mathrm{Tr}[\Phi\Phi]$ and $\mathrm{Tr}^2[\Phi]$ is described by $\mathrm{Tr}[\Phi\Phi]/\mathrm{Tr}^2[\Phi] = 1 - 2c_1c_3/c_2^2$, and a ratio $\det\Phi/\mathrm{Tr}^3[\Phi]$ is given by $\det\Phi/\mathrm{Tr}^3[\Phi] = c_0 {c_3}^2/c_2^3$. We should note that the superpotential (1.2) does not include any explicit flavor symmetry breaking parameter. The most distinctive feature of the present model is that the U(3) flavor symmetry is spontaneously and completely broken by the non-zero and non-degenerate VEVs of $\langle\Phi\rangle$, without passing any subgroup of U(3). (For example, differently from the present model, a U(3)-nonet scalar Φ in Ref.[6] is broken, not directly, but via a discrete symmetry S_4.)

The idea mentioned above is very attractive, because the model does not include conventional Yukawa coupling constants which explicitly break the flavor symmetry. However, a straightforward application of the model (1.2) needs, at least, four different Φ fields, i.e. Φ_f ($f = u, d, \nu, e$), because we know that mass spectra in the four sectors are completely different from each other. From the economical point of view in a unification model of quarks and leptons, we want that the Φ fields are, at most, two, i.e. Φ_u and Φ_d, which couple to the up-quark and neutrino sectors and to the down-quark and charged lepton sectors, respectively.

For this problem, we have a possible solution: we assume that the charged lepton masses m_{ei} are given by a bilinear mass formula

$$m_{ei} = m_0^e z_i^2, \tag{1.8}$$

while the down-quark masses m_{di} are given by a form

$$m_{di} = m_0^d (z_i^2 + \eta z_i), \tag{1.9}$$

where $z_i = v_i/\sqrt{v_i^2 + v_2^2 + v_3^2}$ and $v_i \delta_{ij} = \langle (\Phi_d)_{ij} \rangle$ in the diagonal basis of $\langle \Phi_d \rangle$. The values of the quark mass ratios $m_d/m_s = 0.050$ and $m_s/m_b = 0.031$ at $\mu = M_Z$ [7] lead to $\eta \simeq -0.11$ and $\eta \simeq -0.13$, respectively, so that we can understand the observed ratios by taking $\eta \simeq -0.12$ within one sigma deviation.[1] (Here, in estimating the value η, we have used the values z_i which are obtained from the pole mass values of the charged leptons, because the ratios are insensitive to the energy scale μ.)

In the present paper, at the outset, we will begin to investigate the down-quark and charged lepton sectors. In the next section, Sec.2, we give a framework of the model, and we will investigate the relations for $\mathrm{Tr}[\Phi\Phi]/\mathrm{Tr}^2[\Phi]$, $\det\Phi/\mathrm{Tr}^3[\Phi]$ and η. Those quantities are essentially described by four parameters. In Sec.3, we will give a speculation in order to obtain explicit values of those parameters, although it is only a toy mode and it should not be seriously taken. Finally, Sec.4 will be devoted to concluding remarks.

2 Model

We assume the following superpotential:

$$W_\Phi = \lambda_\phi \mathrm{Tr}[\Phi\Phi\Phi] + m_{\phi\phi}\mathrm{Tr}[\Phi\Phi] + \mu_\phi^2 \mathrm{Tr}[\Phi] + \lambda_e \mathrm{Tr}[\Phi\Phi Y_e] + m_{ee}\mathrm{Tr}[Y_e Y_e]$$
$$+\lambda_d \mathrm{Tr}[\Phi\Phi Y_d] + m_{dd}\mathrm{Tr}[Y_d Y_d] + m_{d\phi}\mathrm{Tr}[Y_d \Phi], \tag{2.1}$$

where Φ, Y_e and Y_d are U(3)-flavor nonet superfields (for convenience, we denote Φ_d as Φ simply), and the $m_{d\phi}$-term has been added in order to give the down-quark mass formula (1.9). In order to couple Y_e and Y_d with the charged lepton sector LEH_d and down-quark sector QDH_d, respectively, for example, we may assign additional U(1) charges $(q_e, -q_e)$ and $(q_d, -q_d)$ to the fields (Y_e, E) and (Y_d, D), respectively. However, such U(1) charges cannot be conserved in W_Φ unless the U(1) charges are also suitably assigned to the coefficients in W_Φ. For the moment, we assume such phenomenological assignments of the U(1) charges to the coefficients in W_Φ.

From the SUSY vacuum conditions, we obtain

$$\frac{\partial W}{\partial \Phi} = 0 = 3\lambda_\phi \Phi\Phi + 2m_{\phi\phi}\Phi + \mu_\phi^2 \mathbf{1} + \lambda_e(\Phi Y_e + Y_e\Phi) + \lambda_d(\Phi Y_d + Y_d \Phi) + m_{d\phi} Y_d, \tag{2.2}$$

[1] However, this possibility is still controversial. Recent updated quark mass estimates [8] and [9] have reported $(m_d/m_s = 0.051; m_s/m_b = 0.019)$ and $(m_d/m_s = 0.052; m_s/m_b = 0.017)$, respectively, as the values at $\mu = M_{GUT} = 2\times 10^{16}$ GeV with $\tan\beta = 10$. Although the both values of m_d/m_s lead to $\eta \simeq -0.11$, the values $m_s/m_b = 0.019$ and $m_s/m_b = 0.017$ lead to $\eta \simeq -0.17$ and $\eta \simeq -0.18$, respectively, so that the unified description based on Eq.(1.9) fails. However, note that the quark mass values are highly dependent on the value of $\tan\beta$. Besides, we do not always consider that the relations (1.8) and (1.9) are given at $\mu = M_{GUT}$. The mass ratio m_s/m_b is highly dependent on the energy scale μ. Therefore, in the present paper, by considering that the scenario (1.9) is applicable to the observed quark masses, we will go on investigating.

$$\frac{\partial W}{\partial Y_e} = 0 = \lambda_e \Phi\Phi + 2m_{ee} Y_e, \tag{2.3}$$

$$\frac{\partial W}{\partial Y_d} = 0 = \lambda_d \Phi\Phi + 2m_{dd} Y_d + m_{d\phi}\Phi, \tag{2.4}$$

so that the mass formula (1.8) and (1.9) are realized by

$$\langle Y_e \rangle = -\frac{\lambda_e}{2m_{ee}} \langle \Phi \rangle \langle \Phi \rangle, \tag{2.5}$$

$$\langle Y_d \rangle = -\frac{\lambda_d}{2m_{dd}} \left(\langle \Phi \rangle \langle \Phi \rangle + \frac{m_{d\phi}}{\lambda_d} \langle \Phi \rangle \right), \tag{2.6}$$

respectively. (Hereafter, for simplicity, we will denote $\langle \Phi \rangle$, $\langle Y_e \rangle$ and $\langle Y_d \rangle$ as Φ, Y_e and Y_d.) By substituting Eqs.(2.5) and (2.6) into (2.2), we again obtain the same equation with (1.6),

$$c_3 \Phi\Phi\Phi + c_2 \Phi\Phi + c_1 \Phi + c_0 \mathbf{1} = 0, \tag{2.7}$$

where

$$c_3 = \frac{\lambda_e^2}{m_{ee}} + \frac{\lambda_d^2}{m_{dd}}, \tag{2.8}$$

$$c_2 = -3\lambda_\phi \left(1 - \frac{1}{2} \frac{m_{d\phi}}{m_{dd}} \frac{\lambda_d}{\lambda_\phi} \right), \tag{2.9}$$

$$c_1 = -2m_{\phi\phi} \left(1 - \frac{1}{4} \frac{(m_{d\phi})^2}{m_{\phi\phi} m_{dd}} \right), \tag{2.10}$$

$$c_0 = -\mu_\phi^2, \tag{2.11}$$

and the coefficients c_n have the following relations with Tr[Φ], Tr[$\Phi\Phi$] and detΦ

$$\frac{c_2}{c_3} = -\mathrm{Tr}[\Phi], \tag{2.12}$$

$$\frac{c_1}{c_3} = \frac{1}{2} \left(\mathrm{Tr}^2[\Phi] - \mathrm{Tr}[\Phi\Phi] \right), \tag{2.13}$$

$$\frac{c_0}{c_3} = -\mathrm{det}\Phi. \tag{2.14}$$

Here, it is convenient to define the following parameters:

$$\tilde{m}_{\phi\phi} = \frac{m_{\phi\phi}}{\lambda_\phi^2}, \quad \tilde{m}_{ee} = \frac{m_{ee}}{\lambda_e^2}, \quad \tilde{m}_{dd} = \frac{m_{dd}}{\lambda_d^2}, \quad \tilde{m}_{d\phi} = \frac{m_{d\phi}}{\lambda_\phi \lambda_d}, \quad \tilde{\mu}_\phi^2 = \frac{\mu_\phi^2}{\lambda_\phi^3}, \tag{2.15}$$

$$\alpha = \frac{\tilde{m}_{dd}}{\tilde{m}_{ee}}, \quad \beta = \frac{\tilde{m}_{\phi\phi}}{\tilde{m}_{dd}}, \quad \gamma = \frac{\tilde{m}_{d\phi}}{\tilde{m}_{dd}}, \quad \delta = \frac{\tilde{\mu}_\phi^2}{\tilde{m}_{dd}^2}. \tag{2.16}$$

Note that those parameters α, β, γ and δ are invariant under the scale transformation of the fields $\Phi \to \xi_\phi \Phi$, $Y_e \to \xi_e Y_e$ and $Y_d \to \xi_d Y_d$, because $\tilde{m}_{\phi\phi} \to \tilde{m}_{\phi\phi}/\xi_\phi^4$, $\tilde{m}_{ee} \to \tilde{m}_{ee}/\xi_\phi^4$, $\tilde{m}_{dd} \to \tilde{m}_{dd}/\xi_\phi^4$, $\tilde{m}_{d\phi} \to \tilde{m}_{d\phi}/\xi_\phi^4$ and $\tilde{\mu}^2 \to \tilde{\mu}^2/\xi_\phi^8$. From Eqs.(2.12) and (2.13), we obtain

$$v_1 + v_2 + v_3 = \mathrm{Tr}[\Phi] = \frac{3}{2}\lambda \tilde{m}_{dd} \frac{2-\gamma}{1+\alpha}, \tag{2.17}$$

$$R \equiv \frac{v_1^2 + v_2^2 + v_3^2}{(v_1+v_2+v_3)^2} = \frac{\mathrm{Tr}[\Phi\Phi]}{\mathrm{Tr}^2[\Phi]} = 1 - 2\frac{c_1 c_3}{c_2^2} = 1 - \frac{4}{9}(1+\alpha)\frac{\gamma^2 - 4\beta}{(\gamma-2)^2}. \tag{2.18}$$

The deviation from the bilinear form in the down-quark mass formula [η in the expression (1.9)] are described by

$$\eta = \frac{m_{d\phi}/\lambda_d}{\sqrt{\mathrm{Tr}[\Phi\Phi]}} = -\frac{2}{3}(1+\alpha)\frac{\gamma}{\gamma-2}\left[1 - \frac{4}{9}(1+\alpha)\frac{\gamma^2-4\beta}{(\gamma-2)^2}\right]^{-1/2}, \tag{2.19}$$

from Eq.(2.6). On the other hand, the ratio $\det\Phi/\mathrm{Tr}^3[\Phi]$ is given by

$$r_{123} \equiv \frac{v_1 v_2 v_3}{(v_1+v_2+v_3)^3} = \frac{\det\Phi}{\mathrm{Tr}^3[\Phi]} = \frac{c_0 c_3^2}{c_2^3} = \frac{8}{27}\frac{(1+\alpha)^2}{(2-\gamma)^3}\delta, \tag{2.20}$$

from (2.14).

From Eqs.(1.1), (2.5), (2.6) and (2.17), we can obtain m_0^f ($f = e, d$) defined in Eqs.(1.8) and (1.9) as follows:

$$m_0^f = -\frac{y_f}{M}\frac{\lambda_f}{2m_{ff}} v^2 v_H = -\frac{3}{8}\frac{y_f}{M}\frac{\lambda_f}{m_{ff}}\lambda_\phi^2 \left(\frac{m_{dd}}{\lambda_d^2}\right)^2 \left(\frac{2-\gamma}{1+\alpha}\right)^2 \frac{1}{(z_3+z_2+z_1)^2} v_H, \tag{2.21}$$

where $\langle \Phi_{ii} \rangle = v_i = v z_i$ ($v^2 = v_1^2 + v_2^2 + v_3^2$) and $v_H = \langle H_d^0 \rangle$. Since the order of m_0^f is $m_0^f \sim m_{dd}^2/M m_{ff}$, we can consider $m_{ff}/M \sim 10^{-2}$.

In order to give explicit values of the mass spectra, we need further assumptions. In the present paper, we are interested in the ratio R which is given by (2.18), because if we can give $R = 2/3$, which means VEV relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1+v_2+v_3)^2, \tag{2.22}$$

we can obtain the following mass relation [10] for the charged leptons

$$m_e + m_\mu + m_\tau = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2, \tag{2.23}$$

from the bilinear mass formula (2.5) on the diagonal basis of $\langle\Phi\rangle$.

We consider that the ratio R is a fundamental quantity in the present model, so that we expect that the ratio R will be expressed by a simple form. Since the $m_{d\phi}$-term is an extra term from the point of view of an $e \leftrightarrow d$ symmetry in the superpotential (2.1), we consider on trial that the ratio R will be independent of such a parameter $\gamma = \tilde{m}_{d\phi}/\tilde{m}_{dd}$. This demands

$$R = \lim_{\gamma\to\infty} R = \lim_{\gamma\to 0} R. \tag{2.24}$$

From the requirement (2.24) for the case $\gamma \to \infty$, we obtain the relation

$$\gamma = \beta + 1, \tag{2.25}$$

i.e.

$$\tilde{m}_{d\phi} = \tilde{m}_{dd} + \tilde{m}_{\phi\phi}. \tag{2.26}$$

Then, we get a simple expression

$$R = 1 - \frac{4}{9}(1+\alpha). \tag{2.27}$$

(For $\gamma \to 0$, we take $\beta \to -1$ from the condition (2.25), so that we again obtain the result (2.27).) Note that the ratio R is described only by one parameter $\alpha = \tilde{m}_{dd}/\tilde{m}_{ee}$.

3 Speculations

In the present section, in order to speculate the values of the parameters α and β, let us put the following assumptions on trial:

$$\lambda_e + \lambda_\phi + \lambda_d = 0, \tag{3.1}$$

$$m_{ee} + m_{\phi\phi} + m_{dd} + m_{d\phi} = 0, \tag{3.2}$$

although there is no theoretical ground for such requirements. We consider that the ratio R is a fundamental quantity in the model, so that it is likely that the ratio is rational. Therefore, we consider that the relations (3.1) and (3.2) are also rational. Since the $m_{d\phi}$-term is concerned with Φ and Y_d, we consider that more fundamental parameter will be λ_e rather than λ_ϕ. Therefore, we assume that the relation (3.1) will be expressed rationally in the unit of λ_e, e.g.

$$\lambda_\phi = n\lambda_e, \quad \lambda_d = -(n+1)\lambda_e, \quad (n = 1, 2, \cdots). \tag{3.3}$$

For the relation (3.2) with an additional assumption $m_{d\phi} = m_{dd} + m_{\phi\phi}$ (cf. Eq.(2.26)), i.e. for $m_{ee} + 2m_{\phi\phi} + 2m_{dd} = 0$, we assume requirements similar to (3.3):

$$2m_{\phi\phi} = n m_{ee}, \quad 2m_{dd} = -(n+1)m_{ee}, \quad (n = 1, 2, \cdots). \tag{3.4}$$

Since we define $\mathrm{Tr}[\Phi] > 0$ and we search the solutions with $\eta < 0$, the signs of m_{dd} and $m_{\phi\phi}$ must be opposite each other. By considering the relation $m_{dd} = -(n+1)m_{\phi\phi}/n$ from (3.4), we must take n as $n > 0$. Then, the parameters α and β are given by

$$\alpha = \frac{m_{dd}\lambda_e^2}{m_{ee}\lambda_d^2} = -\frac{1}{2(n+1)}, \quad \beta = \frac{m_{\phi\phi}\lambda_d^2}{m_{dd}\lambda_\phi^2} = -\frac{n+1}{n}, \tag{3.5}$$

and the ratio R and parameter η are given by

$$R = \frac{5n+7}{9(n+1)}, \tag{3.6}$$

$$\eta = -\frac{1}{\sqrt{(n+1)(5n+7)}}. \tag{3.7}$$

We assume that the ratio R should be as large as possible. This demands $n = 1$ in Eq.(3.6) with $n = 1, 2, \cdots$. (On the other hand, the case $n = 1$ gives a minimum of $|\eta|$.) Then, we can obtain $\alpha = -1/4$ which predicts the desirable relation

$$R = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \tag{3.8}$$

and we can predict a value of η

$$\eta = -\frac{1}{\sqrt{24}} = -0.204. \tag{3.9}$$

Regrettably, the magnitude of the predicted value (3.9) is somewhat larger than the desirable value $\eta \simeq -0.12$ which is estimated from the formula (1.9) with the observed quark mass ratios [7]. However, we do not consider that this discrepancy (3.9) in the present toy model denies the basic idea suggested in Sec.2.[2] Rather, we consider that the order of the value (3.9) is reasonable, so that our direction is basically right.

Finally, let us speculate the value of the ratio r_{123}. From Eq.(2.20), we obtain

$$r_{123} = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{2}{27} \frac{n^3}{(n+1)^2(2n+1)} \delta. \tag{3.10}$$

Since the value r_{123} should be zero in the limit $m_e \to 0$, we expect that the value is realized as small as possible. This again demands $n = 1$ in (3.10), and we obtain

$$r_{123} = \frac{1}{2 \cdot 3^4} \delta. \tag{3.11}$$

Previously, we have assumed the constraints (3.1) and (3.2) for the quadratic and cubic terms in the superpotential (2.1), while, for the tadpole term, since the tadpole term is the μ_ϕ^2-term alone, we cannot put such a speculative relation, so that we cannot speculate a value of δ. The value δ is completely free, although we consider that the value is also rational. Therefore, we give

[2] In order to adjust the predicted values of m_{di}, for example, we may add a tadpole term $\mu_d^2 \mathrm{Tr}[Y_d]$ to the superpotential (2.1). Then, the down-quark mass spectrum will be given by $m_{di} = m_0^d (z_i^2 + \eta_2 z_i + \eta_0)$ instead of (1.9). However, such an additional term will affect the coefficient c_1 defined in Eq.(2.7). From the point of view of simplicity, in the present paper, we do not consider such a modification.

up the prediction of the value r_{123}, and instead, we estimate of a value of δ from the observed charged lepton mass rations. The observed charged lepton masses [11] give

$$z_1 = 0.01647, \quad z_2 = 0.23687, \quad z_3 = 0.97140, \tag{3.12}$$

as the values of $z_i = \sqrt{m_{ei}/(m_e + m_\mu + m_\tau)}$. Although we know that above values are excellently satisfy the relation (2.23) [i.e. (3.8)], the "masses" in the present model mean not "pole" masses, but the "running" masses. For example, if we adopt the mass values [9] at $\mu = 2 \times 10^{16}$ GeV which are estimated from a SUSY scenario with $\tan\beta = 10$, we obtain

$$z_1 = 0.01619, \quad z_2 = 0.23517, \quad z_3 = 0.97182. \tag{3.13}$$

The values (3.12) and (3.13) well satisfy the relation (3.8), i.e. within the deviation 2×10^{-6} and 3×10^{-3}, respectively. However, for the ratio $r_{123} = z_1 z_2 z_3/(z_1 + z_2 + z_3)^3$, both values give slightly different values of r_{123}: the values (3.12) give $r_{123} = 0.002063$, while the values (3.13) give $r_{123} = 0.002013$. (Besides, the value r_{123} is considerably dependent on the value of $\tan\beta$.) In the present paper, we ignore such a small difference. Since we consider the parameter δ will be also expressed with a concise rational value, we take it on trial as

$$\delta = \frac{1}{3}. \tag{3.14}$$

Then, we obtain

$$r_{123} = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{1}{2 \cdot 3^5} = 0.002058. \tag{3.15}$$

so that we obtain the predicted values of z_i, $z_1 = 0.01642$, $z_2 = 0.2369$ and $z_3 = 0.97139$, which are in good agreement with (3.12) [and also (3.13)]. The rational value $\delta = 1/3$ is plausible, although we have no theoretical ground for $\delta = 1/3$.

4 Concluding remarks

In conclusion, we have investigated the charged lepton and down-quark mass spectra on the basis of a model in which the quark and lepton mass spectra originate not in structures of Yukawa coupling constants, but in structures of VEVs of U(3)-flavor nonet (gauge singlet) fields Φ_u and Φ_d. We have proposed a mechanism which gives a bilinear form $M_e \propto \langle\Phi_d\rangle\langle\Phi_d\rangle$ for the charged lepton mass matrix M_e, and which gives a form $M_d \propto \langle\Phi_d\rangle\langle\Phi_d\rangle + c\langle\Phi_d\rangle$ for the down-quark mass matrix M_d. The U(3)-flavor symmetry is spontaneously and completely broken without passing any subgroup of U(3), i.e. directly. The VEV spectrum $\langle\Phi_d\rangle = v\,\mathrm{diag}(z_1, z_2, z_3)$ is completely determined by the coefficients in the superpotential W_Φ, (2.1). The superpotential (2.1) does not include any symmetry breaking term. As shown in Sec.2, the VEV spectrum of $\langle\Phi_d\rangle$ (we have denoted Φ_d as Φ simply) is essentially described by four parameters which have been defined in (2.16). Thus, the VEV spectrum is closely related to the both parameters in the charged lepton and down-quark sectors. The rations $R = (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$ and $r_{123} = \sqrt{m_e m_\mu m_\tau}/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3$ are given by Eqs.(2.18) and (2.20), respectively. The deviation parameter η from the bilinear form $\langle\Phi_d\rangle^2$ in the down-quark mass formula (1.9) is given by (2.19). Those observable quantities are described by the parameters α, β, γ and δ which are invariant under the scale transformations $\Phi \to \xi_\phi \Phi$, $Y_e \to \xi_e Y_e$ and $Y_d \to \xi_d Y_d$.

In order to reduce the number of the parameters, we have assumed that the ratio R is independent of the parameter $\gamma = \tilde{m}_{d\phi}/\tilde{m}_{dd}$ and we have gotten the constraint $\gamma = \beta + 1$. Then, we have obtained a simple expression of R, $R = 1 - 4(1+\alpha)/9$, so that the ratio R is given only by the parameter $\alpha = \tilde{m}_{dd}/\tilde{m}_{ee}$.

In order to predict the charged lepton and down-quark masses, we need explicit values of the parameters α, β, γ and δ. In Sec.3, we have demonstrated a trial scenario to obtain those parameters. By assuming that these parameters have rational relations, we have obtained $\alpha = -1/4$ and $\beta = -2$. The result $\alpha = -1/4$ predicts the desirable relation $R = (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$, while the value $\beta = -2$ predicts $\eta \simeq -0.20$, which is somewhat deviated from the value $\eta \simeq -0.12$ estimated from the observed values of m_d/m_s and m_s/m_b. In order to fit the ratio $r_{123} = \sqrt{m_e m_\mu m_\tau}/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3$, a concise rational value $\delta = 1/3$ is required. Such a value $\delta = 1/3$ seems to be plausible. However, the scenario given in Sec.3 is highly speculative, so that it should not be taken seriously. How to get those parameter values more naturally is a future task to us. However, as we have demonstrated in Sec.3, the idea that those parameter values are described by simple and rational numbers seems to be promising.

By the way, in the present model, we have assumed that the nonet field Φ is Hermitian. Therefore, the parameters in the superpotential have been taken real. This does not mean that all components of $\langle\Phi\rangle$ are real, although three eigenvalues are real. Therefore, the model can include a CP-violating phase. However, in order to give such a phase explicitly, we will need further modification to the superpotential form (2.1).

In the present paper, we have not investigated the up-quark and neutrino mass matrices. We suppose that the up-quark and neutrino mass spectra are given by a similar mechanism for another nonet field Φ_u. (There is a possibility that the observed up-quark and neutrino (Dirac) mass spectra are also described by the forms $m_{ui} \propto (z_{ui})^2 + \eta z_{ui}$ and $m_{\nu i}^{Dirac} \propto (z_{ui})^2$ if we assume a VEV spectrum $z_{ui}\delta_{ij} = \langle(\Phi_u)_{ij}\rangle/\sqrt{\mathrm{Tr}[\Phi_u \Phi_u]}$ different from $\langle\Phi_d\rangle$.) However, the present formulation is applicable only to mass spectra. In order to give non-trivial flavor mixings, the diagonal bases of $\langle\Phi_u\rangle$ and $\langle\Phi_d\rangle$ must be different from each other. If we have a superpotential W which consists of two sets $W_u(\Phi_u, Y_u, Y_\nu)$ and $W_d(\Phi_d, Y_d, Y_e)$ and in which there are no cross terms between (Φ_u, Y_u, Y_ν) and (Φ_d, Y_d, Y_e), we can take two different bases, $\langle\Phi_u\rangle$-diagonal basis and $\langle\Phi_d\rangle$-diagonal basis, separately. However, since, in such a model, there are no parameters which describe relations between $\langle\Phi_u\rangle$ and $\langle\Phi_d\rangle$, we cannot predict the mixing matrices. As we stressed in Sec.1, the most distinctive feature of the present model is that the scenario does not include any explicit flavor symmetry breaking parameter. However, in order to give an explicit relation between $\langle\Phi_u\rangle$ and $\langle\Phi_d\rangle$, we will be obliged to introduce some symmetry breaking term as a flavor-basis fixing term (for example, see Ref.[12]).

The idea that the flavor mass spectra originate in a VEV structure of a U(3)-nonet scalar seems to be promising for unified understanding of quark and lepton masses and mixings.

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