Charged Lepton Mass Relations in a Supersymmetric Yukawaon Model

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Abstract

According to an idea that effective Yukawa coupling constants Y_f^{eff} are given vacuum expectation values $\langle Y_f \rangle$ of fields ("Yukawaons") Y_f as $Y_f^{eff} = y_f \langle Y_f \rangle / \Lambda$, a possible superpotential form in the charged lepton sector under a U(3) [or O(3)] flavor symmetry is investigated. It is found that a specific form of the superpotential can lead to an empirical charged lepton mass relation without any adjustable parameters.

1 Introduction

The so-called "Yukawaon model" [for example, see Ref.[1]] claims that, in effective Yukawa interactions of quarks and leptons

$$H_Y = \sum_{i,j} \ell_i (Y_e^{eff})_{ij} e_j^c H_d + \cdots , \tag{1.1}$$

the effective Yukawa coupling constants Y_f^{eff} $(f = e, \nu, u, d)$ are given by the vacuum expectation values (VEVs) $\langle Y_f \rangle$ of a scalar field Y_f as

$$Y_f^{eff} = \frac{y_f}{\Lambda} \langle Y_f \rangle . \tag{1.2}$$

Here, for simplicity, we have explicitly denoted only the charged lepton sector. In Eq.(1.1), ℓ and e^c are $SU(2)_L$ doublet and singlet fields, respectively, and Λ is an energy scale of the effective theory. (We have considered a supersymmetric (SUSY) scenario.) Hereafter, we refer the fields Y_f as "Yukawaons" [1], which are gauge singlets. In addition to the Yukawaon Y_e, we consider a field Φ_e which is related to Y_e as

$$\langle Y_e \rangle = k \langle \Phi_e \rangle \langle \Phi_e \rangle . \tag{1.3}$$

We also refer Φ_e as a "ur-Yukawaon", which has been introduced in order to fix the VEVs of the Yukawaon Y_e. (For the moment, we consider the ur-Yukawaon only in the charged lepton sector.) Then, an empirical charged lepton mass formula [2]

$$R_e \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \tag{1.4}$$

is rewritten as

$$R_e \equiv \frac{v_1^2 + v_2^2 + v_3^2}{(v_1 + v_2 + v_3)^2} = \frac{2}{3}, \tag{1.5}$$

where $v_i = \langle (\Phi_e)_{ii} \rangle$.

Previously, the author [3] has derived the relation (1.5) by assuming the following U(3)-flavor-invariant scalar potential

$$V = \mu^2(\pi^2 + \eta^2 + \sigma^2) + \lambda(\pi^2 + \eta^2 + \sigma^2)^2 + \lambda'(\pi^2 + \eta^2)\sigma^2, \tag{1.6}$$

where

$$\pi = \frac{1}{\sqrt{2}}(\Phi_{11} - \Phi_{22}), \quad \eta = \frac{1}{\sqrt{6}}(\Phi_{11} + \Phi_{22} - 2\Phi_{33}), \quad \sigma = \frac{1}{\sqrt{3}}(\Phi_{11} + \Phi_{22} + \Phi_{33}), \tag{1.7}$$

and $\pi^2 + \eta^2 + \sigma^2$ and σ^2 correspond to $\mathrm{Tr}[\Phi\Phi]$ and $\frac{1}{3}\mathrm{Tr}^2[\Phi]$ in a diagonal basis of the VEV matrix $\langle \Phi \rangle$, respectively. Here, we have dropped the index "e" in Φ_e for convenience. Since, in the present paper, we often meet with traces of matrices A, hereafter, we denote the traces $\mathrm{Tr}[A]$ as $[A]$ concisely. The scalar potential (1.6) can be rewritten as

$$V = \mu^2[\Phi\Phi] + \lambda[\Phi\Phi]^2 + \frac{1}{3}\lambda'[\Phi^{(8)}\Phi^{(8)}][\Phi]^2, \tag{1.8}$$

where $\Phi^{(8)}$ is an octet part of the nonet field Φ, $\Phi^{(8)} = \Phi - \frac{1}{3}[\Phi]$. The minimizing condition of V demands

$$\frac{\partial V}{\partial \Phi} = 2\left(\mu^2 + 2\lambda[\Phi\Phi] + \frac{1}{3}\lambda'[\Phi]^2\right)\Phi + \frac{2}{3}\lambda'\left([\Phi\Phi] - \frac{2}{3}[\Phi]^2\right)[\Phi] = 0, \tag{1.9}$$

so that we obtain the relation (1.5), i.e.

$$R = \frac{[\Phi\Phi]}{[\Phi]^2} = \frac{2}{3}, \tag{1.10}$$

together with $\mu^2 + 2\lambda[\Phi\Phi] + \frac{1}{3}\lambda'[\Phi] = 0$. Of course, a statement that the relation (1.10) was derived by assuming U(3) symmetry is not correct. The accurate statement is that the relation (1.10) was derived from a scalar potential (1.6) [(1.8)] which is invariant under U(3) symmetry, but which is not a general form of the U(3) invariant scalar potential.

A straightforward SUSY version of the scalar potential (1.8) is as follows: the superpotential W is given by

$$W = \mu[\Phi A] + \lambda'[\Phi][\Phi^{(8)}B], \tag{1.11}$$

where A and B are additional nonet fields. Then, the superpotential (1.11) leads to a scalar potential

$$V = |\mu|^2[\Phi\Phi^\dagger] + |\lambda'|^2[\Phi][\Phi]^\dagger[\Phi^{(8)}\Phi^{(8)\dagger}] + \cdots . \tag{1.12}$$

However, although the minimizing condition of the scalar potential (1.12) can lead to the relation (1.10), the vacuum is not stable.

A supersymmetric approach with SUSY vacuum conditions to the mass relation (1.4) has first been done by Ma [4]. His model with a flavor symmetry $\Sigma(81)$ is impeccable, but somewhat intricate. Stimulated by his work, the author [5] has also proposed a superpotential with a simple form

$$W = \mu[\Phi\Phi] + \lambda[\Phi\Phi\Phi], \tag{1.13}$$

by assuming a Z_2 symmetry in addition to the U(3) flavor symmetry. Here, it has been assumed that the octet $\Phi^{(8)} = \Phi - \frac{1}{3}[\Phi]$ and singlet $\Phi^{(1)} = \frac{1}{3}[\Phi]$ have Z_2 parities -1 and $+1$, respectively. Then, in the cubic term

$$[\Phi\Phi\Phi] = [\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] + [\Phi^{(8)}\Phi^{(8)}][\Phi] + \frac{1}{3}[\Phi^{(8)}][\Phi]^2 + \frac{1}{9}[\Phi]^3, \tag{1.14}$$

the Z_2 parities of the terms $[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}]$ and $[\Phi^{(8)}][\Phi]^2$ are -1, so that those terms are dropped under the Z_2 symmetry:

$$[\Phi\Phi\Phi]_{Z_2=+1} = [\Phi^{(8)}\Phi^{(8)}][\Phi] + \frac{1}{9}[\Phi]^3 = [\Phi][\Phi\Phi] - \frac{2}{9}[\Phi]^3. \tag{1.15}$$

Then, by requiring a SUSY vacuum condition

$$\frac{\partial W}{\partial \Phi} = 2(\mu + \lambda[\Phi])\Phi + \lambda\left([\Phi\Phi] - \frac{2}{3}[\Phi]^2\right)[\Phi] = 0, \tag{1.16}$$

where we have used $[\Phi^{(8)}\Phi^{(8)}] = [\Phi\Phi] - \frac{1}{3}[\Phi]^2$, we can obtain the relation (1.10). However, such the Z_2 charge assignment requires a somewhat intricate scenario [5] when Φ is related to Y, because we need not only $\Phi^{(8)}\Phi^{(8)} + \Phi^{(1)}\Phi^{(1)}$ with $Z_2 = +1$, but also $\Phi^{(8)}\Phi^{(1)} + \Phi^{(1)}\Phi^{(8)}$ with $Z_2 = -1$ in $Y = k\Phi\Phi$.

If we accept a higher dimensional term in the superpotential, by assuming a simple form without such Z_2 symmetry

$$W = \mu[\Phi\Phi] + \frac{1}{\Lambda}[\Phi]^2[\Phi^{(8)}\Phi^{(8)}], \tag{1.17}$$

we can also obtain the relation (1.10):

$$\frac{\partial W}{\partial \Phi} = 2\left(\mu + \frac{1}{\Lambda}[\Phi]^2\right)\Phi + \frac{2}{\Lambda}\left([\Phi\Phi] - \frac{2}{3}[\Phi]^2\mathbf{1}\right)[\Phi] = 0. \tag{1.18}$$

However, we must recall that each Yukawaon Y_f has a different $U(1)_X$ charge $Q_X = x_f$ in order to distinguish each fermion partner [6]. Since the ur-Yukawaon Φ_e also has a $U(1)_X$ charge $Q_X = \frac{1}{2}x_e$, we cannot write the superpotential (1.17) [also Eq.(1.13)] without violating the $U(1)_X$ symmetry.

We would like to search for a superpotential form whose vacuum conditions lead to the relation (1.10) under the conditions that (i) the superpotential W does not include a higher dimensional term, and (ii) W is invariant under the U(3) [or O(3)] and $U(1)_X$ symmetries. Note that, in the original idea (1.6), the result (1.10) is obtained independently of the explicit parameter values μ, λ and λ'. We consider that such a motive should be inherited in a SUSY version of the scenario, too. The result (1.10) should be obtained without adjusting parameters in the model. We will search for a superpotential form by considering that the form may include an ad hoc term for the time being, but the form should be simple.

2 Ansatz and VEV relations

In the present paper, we assume the Yukawaons Y_f are nonets of a U(3)-flavor symmetry [or $\mathbf{5}+\mathbf{1}$ of O(3)], and those do not solely appear as octets of U(3) [or $\mathbf{5}$-plets of O(3)] in the superpotential. On the other hand, as suggested by the forms (1.11) and (1.17), the traceless part of Φ_e, $\hat{\Phi}_e \equiv \Phi_e - \frac{1}{3}[\Phi_e]$, seems to play an crucial role in obtaining the relation (1.10). Therefore, for the ur-Yukawaon Φ_e, we consider that the traceless part $\hat{\Phi}_e$ of the ur-Yukawaon can solely appear in the superpotential.

In order to obtain a bilinear relation

$$Y_e = k\Phi_e\Phi_e, \tag{2.1}$$

we assume a superpotential term [6]

$$W_A = \lambda_A[\Phi_e\Phi_e A_e] + \mu_A[Y_e A_e], \tag{2.2}$$

where $k = -\lambda_A/\mu_A$ and these fields have U(1)$_X$ charges as $Q_X(Y_e) = x_e$, $Q_X(\Phi_e) = \frac{1}{2}x_e$ and $Q_X(A_e) = -x_e$. In addition to the field A_e, we introduce a new field A'_e which couples only to $\hat{\Phi}_e$ as $[\hat{\Phi}_e\hat{\Phi}_e A'_e]$, and we also introduce a field Y'_e which composes a mass term $\mu''[Y'_e A'_e]$ together with A'_e similarly to Eq.(2.2). Since the new field A'_e has the same U(1)$_X$ charge with A_e, we can write the superpotential as follows:

$$W_A = \lambda_A[\Phi_e\Phi_e A_e] + \mu_A[Y_e A_e] + \lambda'_A[\Phi_e\Phi_e A'_e] + \mu'_A[Y_e A'_e]$$
$$+\lambda''_A[\hat{\Phi}_e\hat{\Phi}_e A'_e] + \lambda'''_A\phi_x[Y'_e A'_e], \tag{2.3}$$

where $Q_X(A'_e) = -x_e$, $Q_X(\phi_x) = x_\phi$ and $Q_X(Y'_e) = x_e - x_\phi$. Here, the reason that we have written $\lambda'''_A\phi_x$ instead of μ''_A in Eq.(2.3) is to distinguish Y'_e from Y_e in order to prevent $(Y'_e)_{ij}$ from coupling with $\ell_i e^c_j$. In general, when fields A_1 and A_2 with the same U(1)$_X$ charges couple with four terms $\Phi_e\Phi_e$, Y_e, $\hat{\Phi}_e\hat{\Phi}_e$ and Y'_e in Eq.(2.3), one of those, for example, $[\hat{\Phi}_e\hat{\Phi}_e(c_1A_1 + c_2A_2)]$, can be rewritten as $\sqrt{c_1^2+c_2^2}\,[\hat{\Phi}_e\hat{\Phi}_e A'_e]$ without losing generality. Therefore, the λ''_A-term in Eq.(2.3) is not an ansatz. However, the 6th term (λ'''_A-term) in Eq.(2.3) is, in general, given by a linear combination of A_e and A'_e. Nevertheless, we have defined Y'_e as the field Y'_e can make a mass term only with A'_e. This is just an ansatz in the present scenario. From the SUSY vacuum condition $\partial W/\partial A_e = 0$, and $\partial W/\partial A'_e = 0$, we obtain the VEV relations (2.1) with $k = -\lambda_A/\mu_A$ and

$$Y'_e = -\frac{1}{\lambda'''_A\phi_x}\left(\lambda'_A\Phi_e\Phi_e + \lambda''_A\hat{\Phi}_e\hat{\Phi}_e + \mu'_A Y_e\right), \tag{2.4}$$

respectively. By substituting Eq.(2.1) for (2.4), we obtain a VEV relation

$$Y'_e = k'(\Phi_e\Phi_e + \xi\hat{\Phi}_e\hat{\Phi}_e), \tag{2.5}$$

where

$$k' = -\frac{1}{\lambda'''_A \phi_x}\left(\lambda'_A - \frac{\mu'_A}{\mu_A}\lambda_A\right), \quad \xi = \frac{\lambda''_A}{\lambda'_A - \frac{\mu'_A}{\mu_A}\lambda_A}. \tag{2.6}$$

(The other SUSY vacuum conditions $\partial W/\partial Y_e = 0$, $\partial W/\partial Y'_e = 0$, $\partial W/\partial \phi_x = 0$ and $\partial W/\partial \Phi_e = 0$ lead to $A_e = A'_e = 0$ for $\phi_x \neq 0$.)

Next, we introduce a field B_e with $Q_X = -\frac{3}{2}x_e + x_\phi$, and we write a superpotential term

$$W_B = \lambda_B[\Phi_e Y'_e B_e]. \tag{2.7}$$

The SUSY vacuum condition $\partial W/\partial B_e = 0$ ($W = W_A + W_B$) gives $\Phi_e Y'_e = 0$, i.e.

$$\Phi_e(\Phi_e\Phi_e + \xi\hat{\Phi}_e\hat{\Phi}_e) = (1+\xi)\Phi_e^3 - \frac{2}{3}\xi[\Phi_e]\,\Phi_e^2 + \frac{1}{9}\xi[\Phi_e]^2\,\Phi_e = 0, \tag{2.8}$$

from Eq.(2.5). On the other hand, in general, in a cubic equation

$$\Phi^3 + c_2\Phi^2 + c_1\Phi + c_0\mathbf{1} = 0, \tag{2.9}$$

the coefficients c_i have the following relations:

$$c_2 = -[\Phi], \quad c_1 = \frac{1}{2}\left([\Phi]^2 - [\Phi\Phi]\right), \quad c_0 = -\det\Phi. \tag{2.10}$$

In order that there is a solution $[\Phi_e] \neq 0$, we must take

$$\xi = -3, \tag{2.11}$$

in the coefficient c_2. Then, we obtain the ratio R_e defined by Eq.(1.5) as follows: from the coefficient c_1, we have a relation

$$c_1 = \frac{\xi}{9(1+\xi)}[\Phi_e]^2 = \frac{1}{2}\left([\Phi_e]^2 - [\Phi_e\Phi_e]\right), \tag{2.12}$$

so that we can obtain the ratio

$$R_e \equiv \frac{[\Phi_e\Phi_e]}{[\Phi_e]^2} = 1 - \frac{2\xi}{9(1+\xi)} = \frac{2}{3}, \tag{2.13}$$

by using Eq.(2.11).

Although the present model can give a reasonable value of R_e, the cubic equation (2.8) gives $c_0 = -\det\Phi_e = 0$, which means that the electron is massless, $m_e = 0$. Therefore, next, we are interested in the following ratio [7]

$$r_{123} = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{\det\Phi_e}{[\Phi_e]^3}, \tag{2.14}$$

whose limit $r_{123} \to 1$ means that the electron is massless. A simple way to obtain a non-vanishing c_0 without affecting the values of c_1 and c_2 in the above scenario is to add an ad hoc term

$$\varepsilon_1 \lambda_B [\Phi_e][Y'_e][B_e], \tag{2.15}$$

to the term (2.7) without violating the U(1)$_X$ symmetry. Then, the coefficient c_0 is given by

$$c_0 = \frac{\varepsilon_1}{1+\xi}[\Phi_e]\left((1+\xi)[\Phi_e\Phi_e] - \frac{\xi}{3}[\Phi_e]^2\right). \tag{2.16}$$

By using the relations (2.11) and (2.13), we obtain

$$c_0 = \frac{1}{6}\varepsilon_1[\Phi_e]^3, \tag{2.17}$$

so that we can obtain the ratio

$$r_{123} = -\frac{1}{6}\varepsilon_1, \tag{2.18}$$

by recalling the relation $c_0 = -\det\Phi_e$. If we consider another term

$$\varepsilon_2 \lambda_B [\Phi_e Y'_e][B_e], \tag{2.19}$$

we can also obtain

$$c_0 = 3\varepsilon_2 \det\Phi_e, \tag{2.20}$$

where we have used a formula

$$\det A = \frac{1}{3}[A^3] - \frac{1}{2}[A][A^2] + \frac{1}{6}[A]^3. \tag{2.21}$$

Therefore, when we consider both terms (2.15) and (2.19), we obtain

$$r_{123} = -\frac{\varepsilon_1}{6(1+3\varepsilon_2)}. \tag{2.22}$$

If we assume a traceless field $\hat{B}_e \equiv B_e - \frac{1}{3}[B_e]$ instead of B_e in Eq.(2.7), the case corresponds to the case with $\varepsilon_1 = 0$ and $\varepsilon_2 = -1/3$, and we find that c_0 identically becomes $c_0 = -\det\Phi_e$, so that any value of $\det\Phi_e$ is allowed. Therefore, the case is not so interesting. At present, the parameters ε_1 and ε_2 are free, so that we cannot predict the value of r_{123}.

3 Concluding remarks

In conclusion, we have found a superpotential which can lead to the VEV relation (1.10), $[\Phi_e\Phi_e] = \frac{2}{3}[\Phi_e]^2$. It should be noticed that, although we have assumed a U(3) [or O(3)] flavor symmetry in the present paper, it does not mean that the relation (1.10) was derived by assuming the symmetry. The relation (1.10) was obtained by assuming a specific form (2.3) in the superpotential under the flavor symmetry. In the superpotential (2.3), the existence of the term $\Phi_e\hat{\Phi}_e\hat{\Phi}_e$ plays an crucial role in obtaining the relation (1.10). If all allowed terms under the symmetry were indiscriminately taken into consideration, the model would have become a "parameter physics" as well as conventional mass matrix models. (We have chosen ξ as $\xi = -3$

in Eq.(2.11). However, we do not regard ξ as an adjustable parameter in the present model. The condition (2.11) has been settled by a fundamental requirement that the non-zero VEV $[\Phi]$ should exist. The parameter ξ is not an adjustable parameter in the phenomenological meaning.)

Since we have successfully obtained the relation (1.10) without adjustable parameters, another problem has risen in the present scenario: We know that $R = 2/3$ is valid only for the charged lepton masses, and the observed masses for another sectors do not satisfy $R = 2/3$. For example, the ratio R_u for the up-quark masses is $R_u \simeq 8/9$ [8]. Can we modify the present scenario as it leads to $R_u \simeq 8/9$? At present it seems to be impossible, because there is no adjustable parameter in the present scenario.

By the way, on the basis of a Yukawaon model, an interesting neutrino mass matrix form [6, 9]

$$M_\nu \propto \langle Y_e\rangle \left(\langle Y_e\rangle\langle\Phi_u\rangle + \langle\Phi_u\rangle\langle Y_e\rangle\right)^{-1} \langle Y_e\rangle, \tag{3.1}$$

has been proposed, where the up-quark mass spectrum is given by $\langle Y_u\rangle \propto \langle\Phi_u\rangle\langle\Phi_u\rangle$. The neutrino mass matrix (3.1) can successfully lead to a nearly tribimaximal mixing [11] under an additional phenomenological assumption. In the successful description of M_ν, it is crucial that the Majorana mass matrix of the right-handed neutrinos M_R is given by linear terms of $\sqrt{m_{ui}}$. Therefore, the bilinear form $\langle Y_u\rangle \propto \langle\Phi_u\rangle\langle\Phi_u\rangle$ seems to be valid for the up-quark sector, too.

We also pay attention to the following empirical relation

$$\sqrt{\frac{m_{ui}}{m_{uj}}} \simeq \frac{m_{ei}+m_0}{m_{ej}+m_0}, \tag{3.2}$$

where m_{ui} and m_{ei} are masses of up-quarks and charged leptons. In fact, for example, the value $m_0 = 4.36$ MeV gives the ratios $(m_e+m_0)/(m_\mu+m_0) = 0.0453$ and $(m_\mu+m_0)/(m_\tau+m_0) = 0.0612$ correspondingly to the observed values $\sqrt{m_u/m_c} = 0.0453^{+0.012}_{-0.010}$ and $\sqrt{m_c/m_t} = 0.0600^{+0.0045}_{-0.0047}$, respectively. (Here, we have used quark mass values [10] at $\mu = m_Z$, because the quark mass values at a unification scale are highly dependent on the value of $\tan\beta = v_u/v_d$.)

These facts suggest a possibility that

$$\langle Y_e\rangle \propto \langle\Phi_e\rangle\langle\Phi_e\rangle, \quad \langle\Phi_e\rangle \equiv \langle\Phi_0^e\rangle, \tag{3.3}$$

in the charged lepton sector, while

$$\langle Y_u\rangle \propto \langle\Phi_u\rangle\langle\Phi_u\rangle, \quad \langle\Phi_u\rangle \propto \langle\Phi_0^u\rangle\langle\Phi_0^u\rangle + \varepsilon\mathbf{1}, \tag{3.4}$$

in the up-quark sector, where ur-Yukawaons Φ_0^e and Φ_0^u exactly have the same VEV spectra, but the diagonal bases of $\langle\Phi_0^e\rangle$ and $\langle\Phi_0^u\rangle$ are different from each other.

Therefore, there is a possibility that all quark and lepton mass spectra (in other words, all $\langle Y_f\rangle$) can be described in terms of only two ur-Yukawaons Φ_0^e and Φ_0^u. However, in Ref.[6, 1], where a supersymmetric Yukawaon model has been investigated on the basis of an O(3) flavor symmetry, the down-quark Yukawaon Y_d has not explicitly been discussed. In the O(3) model [6, 1], since it is assumed that the VEVs of Φ_0^e and Φ_0^u are real, the observed CP violating phase in the quark sector must be inevitably included in the down-quark sector. Whether such a unified description is possible or not is dependent on whether a down-quark Yukawaon Y_d can also reasonably be described in terms of Φ_0^e and Φ_0^u. This will be a touchstone of the Yukawaon approach.

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