Testable Deviations from Exact Tribimaximal Mixing

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Abstract

A simple relation $U_{PMNS} = V_{CKM}^\dagger U_{TB}$ between the lepton and quark mixing matrices (U_{PMNS} and V_{CKM}) is speculated under an ansatz that U_{PMNS} becomes an exact tribimaximal mixing U_{TB} in a limit $V_{CKM} = \mathbf{1}$. By using the observed CKM mixing parameters, possible values of neutrino oscillation parameters are estimated: $\sin^2\theta_{13} = 0.024 - 0.028$, $\sin^2 2\theta_{23} = 0.94 - 0.95$ and $\tan^2\theta_{12} = 0.24 - 1.00$ depending on phase conventions of U_{TB}. Those values are testable soon by precision measurements in neutrino oscillation experiments.

1 Introduction

Recently, there has been considerable interest in the magnitude of the neutrino mixing angle θ_{13} ($\nu_e \leftrightarrow \nu_\tau$ mixing angle), because it is a key value not only for checking neutrino mass matrix models, but also for searching CP-violation effects in the lepton sector. (For a review of models for θ_{13}, see, for example, Ref.[1].) Recent observed neutrino oscillation data are in favor of the so-called "tribimaximal mixing" [2] which predicts $\theta_{13} = 0$, $\tan^2\theta_{12} = 1/2$ and $\sin^2 2\theta_{23} = 1$, since the present data yield the values $\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$ [3] and $\sin^2 2\theta_{23} = 1.00_{-0.13}$ [4]. If the angle θ_{13} is exactly zero or negligibly small, the observation of the CP-violation effects in the lepton sector will be hopeless even in future, as far as neutrino oscillation experiments are concerned. On the other hand, recently, Fogli *et al.* [5] have reported a sizable value $\sin^2\theta_{13} = 0.016 \pm 0.010$ (1σ) from a global analysis of neutrino oscillation data.

The tribimaximal lepton mixing is given by the form

$$U_{TB}^0 = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \tag{1}$$

Such a form with beautiful coefficients seems to be understood from a discrete symmetry of flavors [2]. In contrast to the lepton mixing matrix (Pontecorvo-Maki-Nakagawa-Sakata mixing matrix [6]) U_{PMNS}, the observed Cabibbo-Kobayashi-Maskawa [7] (CKM) quark mixing matrix V_{CKM} seems to have no beautiful form with Clebsch-Gordan-like coefficients, and V_{CKM}, rather, looks like nearly $V_{CKM} \simeq \mathbf{1}$. It is unlikely that a theory which exactly leads to the tribimaximal mixing (1) simultaneously gives the CKM mixing matrix with small and complicated mixing values. Therefore, it is interesting to consider a specific case that a theory of flavor symmetry gives $V_{CKM} = \mathbf{1}$ in the limit of $U_{PMNS} = U_{TB}$. We consider that the observed form of the CKM matrix V_{CKM} is due to some additional effects (e.g. symmetry breaking effects for the flavor symmetry). If this is true, then, the observed lepton mixing U_{PMNS} will also deviate from the exact tribimaximal mixing $U_{PMNS} = U_{TB}$ by additional effects which gives the deviation from $V_{CKM} = \mathbf{1}$. (Also see, e.g., Ref.[8] for a possible deviation of U_{PMNS} from a bimaximal mixing (not tribimaximal mixing) related to V_{CKM}.)

Recently, Datta [9] has investigated possible flavor changing neutral current processes using the same assumption that $V_{CKM} = \mathbf{1}$ and $U_{PMNS} = U_{TB}$ in a flavor symmetry limit. By using a specific mass matrix model, he have discussed realistic mixings V_{CKM} and U_{PMNS} caused by a small breaking of the flavor symmetry. Also, Plentinger and Rodejohann [10] have investigated possible deviations from tribimaximal mixing by assuming a special form of the neutrino mass matrix. Furthermore, there are many works which discuss specific mass matrix models from the point of the so-called "quark-lepton-complementarity" [11]. In this paper, however, we start only from putting a simple ansatz stated later (in Eqs.(9) and (10)), without referring to any mass matrix model explicitly.

For convenience of later discussions, we define the tribimaximal mixing by a form

$$U_{TB} = P_L^* U_{TB}^0 P_R, \tag{2}$$

where

$$\begin{aligned} P_L &= \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}), \\ P_R &= \mathrm{diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3}), \end{aligned} \tag{3}$$

by including freedom of the phase convention, although the tribimaximal mixing is conventionally expressed by the form (1). The purpose of the present paper is to speculate a possible form of the lepton mixing matrix U_{PMNS} under the ansatz $V_{CKM} = \mathbf{1} \leftrightarrow U_{PMNS} = U_{TB}$. We show, as stated later, that a natural realization of this ansatz leads to a simple relation

$$U_{PMNS} = V_{CKM}^\dagger U_{TB}. \tag{4}$$

By using the observed CKM mixing parameters, we estimate values of the neutrino oscillation parameters $\sin^2\theta_{13}$, $\tan^2\theta_{12}$ and $\sin^2 2\theta_{23}$, which are defined by

$$\begin{aligned} \sin^2\theta_{13} &\equiv |(U_{PMNS})_{13}|^2, \\ \tan^2\theta_{12} &\equiv |(U_{PMNS})_{12}/(U_{PMNS})_{11}|^2, \\ \sin^2 2\theta_{23} &\equiv 4|(U_{PMNS})_{23}|^2|(U_{PMNS})_{33}|^2. \end{aligned} \tag{5}$$

First, let us give conventions of the mass matrices: the quark and charged lepton mass matrices M_f ($f = u, d, e$) are defined by the mass terms $\overline{f}_L M_f f_R$, so that those are diagonalized as

$$U_{fL}^\dagger M_f U_{fR} = D_f \equiv \mathrm{diag}(m_{f1}, m_{f2}, m_{f3}), \tag{6}$$

and the neutrino (Majorana) mass matrix M_ν is defined by $\overline{\nu}_L M_\nu \nu_L^c$, so that it is diagonalized as

$$U_{\nu L}^\dagger M_\nu U_{\nu L}^* = D_\nu \equiv \mathrm{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}). \tag{7}$$

Therefore, the quark and lepton mixing matrices, V_{CKM} and U_{PMNS}, are given by

$$V_{CKM} = U_{uL}^\dagger U_{dL}, \quad U_{PMNS} = U_{eL}^\dagger U_{\nu L}, \tag{8}$$

respectively. Hereafter, we refer to a flavor basis on which the mass matrix M_f is diagonal (i.e. D_f) as "f-basis". For example, in the u-basis, up-quark, down-quark, charged-lepton and neutrino mass matrices are given by $D_u = U_{uL}^\dagger M_u U_{uR}$, $M_d^{(u)} = U_{uL}^\dagger M_d U_{uR}$, $M_e^{(u)} = U_{uL}^\dagger M_e U_{uR}$ and $M_\nu^{(u)} = U_{uL}^\dagger M_\nu U_{uL}^*$, respectively.

2 Ansatz and speculation

Let us mention an ansatz which leads to the relation (4). We put the following ansatz: In the limit of $U_{dL} \to \mathbf{1}$, the matrix U_{eL} also becomes a unit matrix $\mathbf{1}$, while the matrix U_ν becomes the exact tribimaximal mixing U_{TB} in the limit of $U_{uL} \to \mathbf{1}$. In other words, in the u-basis, the neutrino mass matrix $M_\nu^{(u)} \equiv U_{uL}^\dagger M_\nu U_{uL}^*$ is diagonalized by the exact tribimaximal mixing matrix U_{TB}, i.e.

$$U_{TB}^\dagger M_\nu^{(u)} U_{TB}^* = D_\nu. \tag{9}$$

Here, we have supposed that, in a symmetry limit, i.e. when an origin which causes $V_{CKM} \neq \mathbf{1}$ is switched off, the physical mass matrices M_f become the diagonal forms D_f, while the neutrino mass matrix M_ν becomes a specific form $M_\nu^{(u)}$ defined by (9):

$$(M_u, M_d;\, M_e, M_\nu) \to (D_u, D_d;\, D_e, U_{TB} D_\nu U_{TB}^T). \tag{10}$$

In other words, we consider that a common origin in the down sector causes $D_d \to M_d$ and $D_e \to M_e$, and a common origin in the up sector causes $D_u \to M_u$ and $U_{TB} D_\nu U_{TB}^T \to M_\nu$. Of course, this transformation (10) can not be realized by a flavor-basis transformation, because M_f and D_f are connected by Eqs.(6) and (7). It is well-known that physics at a low-energy is unchanged under any flavor-basis transformation.

The ansatz (9) states that the mixing matrix $U_{\nu L}$ in the neutrino sector, which is defined by $U_{\nu L}^\dagger M_\nu U_{\nu L}^* = D_\nu$, is given by

$$U_{\nu L} = U_{uL} U_{TB}, \tag{11}$$

because $D_\nu = U_{TB}^\dagger M_\nu^{(u)} U_{TB}^* = U_{TB}^\dagger (U_{uL}^\dagger M_\nu U_{uL}^*) U_{TB}^*$. Therefore, the observed lepton mixing matrix U_{PMNS} is given by

$$U_{PMNS} = U_{eL}^\dagger U_{\nu L} = U_{eL}^\dagger U_{uL} U_{TB} = U_{ed} V_{CKM}^\dagger U_{TB}, \tag{12}$$

where U_{ed} is a flavor-basis transformation matrix defined by

$$U_{ed} = U_{eL}^\dagger U_{dL}. \tag{13}$$

(The relation (12) is also derived by using relations $U_{\nu L}^{(u)} = U_{TB}$ and $U_{eL}^{(u)} = U_{uL}^\dagger U_{eL}$ in the u-basis as $U_{PMNS} = U_{eL}^{(u)\dagger} U_{\nu L}^{(u)} = U_{eL}^\dagger U_{uL} U_{TB} = U_{ed} V_{CKM}^\dagger U_{TB}$.) According to this notation, the CKM mixing matrix V_{CKM} is expressed as $V_{CKM} = U_{ud}$. Since $U_{ed} = U_{ue}^\dagger U_{ud} = U_{ue}^\dagger V_{CKM}$, if we consider $U_{ue} = \mathbf{1}$, we obtain $U_{ed} = V_{CKM}$, so that we will obtain $U_{PMNS} = U_{TB}$ from the relation (11). However, such a case $U_{eu} = \mathbf{1}$ is unlikely under our ansatz $U_{eL} \to \mathbf{1}$ in the limit of $U_{dL} \to \mathbf{1}$. Generally speaking, U_{ue} can vary from $U_{ue} = \mathbf{1}$ to $U_{ue} = V_{CKM}$, so that U_{ed} varies from $U_{ed} = V_{CKM}$ to $U_{ed} = \mathbf{1}$ and Eq.(12) varies from $U_{PMNS} = U_{TB}$ to $U_{PMNS} = V_{CKM}^\dagger U_{TB}$. (Here, we have considered that U_{ue} does, at least, not take a large mixing more than V_{CKM} and a rotation to an opposite direction, $V_{CKM}^\dagger$.) Therefore, we can consider that the relation (4) describes a maximal deviation of U_{PMNS} from U_{TB}. In spite of such a general consideration, we think that the case $U_{ed} = \mathbf{1}$ (or highly $U_{ed} \simeq \mathbf{1}$) is a most natural realization of our ansatz (10), because it means $U_{eL} \to \mathbf{1}$ in the limit $U_{dL} \to \mathbf{1}$. Therefore, in this paper, we adopt the case $U_{ed} = \mathbf{1}$, and investigate possible numerical values of the neutrino oscillation parameters $\sin^2\theta_{13}$, $\tan^2\theta_{12}$ and $\sin^2 2\theta_{23}$ under the relation (4).

By the way, we are also interested in whether those values are dependent on the phase parameters α_i and γ_i defined in Eq.(3). The relation (12) is invariant under the rephasing $U_{fL} \to U_{fL}P_f$ ($f = u, d, e$) because of $V_{CKM} \to P_u^* V_{CKM} P_d$, $U_{PMNS} \to P_e^* U_{PMNS}$, $U_{ed} \to P_e^* U_{ed} P_d$ and $U_{TB} \to P_u^* U_{TB}$ under the rephasing (note that $U_{\nu L}$ does not have such a freedom of rephasing). Therefore, the phase matrices P_L and P_R originate in the mass matrix $M_\nu^{(u)}$ as shown in Eq.(9). Then, Eq.(9) can be rewritten as

$$(U_{TB}^0)^T \widetilde{M}_\nu^{(u)} U_{TB}^0 = D_\nu P_R^2, \tag{14}$$

where

$$\widetilde{M}_\nu^{(u)} = P_L M_\nu^{(u)} P_L. \tag{15}$$

Since the matrix U_{TB}^0 is orthogonal, the mass matrix $\widetilde{M}_\nu^{(u)}$ has to be real. In other words, the phase matrix P_L is determined from the form $M_\nu^{(u)}$ so that $\widetilde{M}_\nu^{(u)}$ is real. On the other hand, the phase matrix P_R is fixed so that $D_\nu P_R^2$ is real. Then, we find that the numerical results for $|(U_{PMNS})_{ij}|$ are independent of the phases γ_i in P_R, because U_{PMNS} is expressed by $U_{PMNS} = U_{PMNS}^{P_R=\mathbf{1}} P_R$, so that the quantities $|(U_{PMNS})_{ij}| = |(U_{PMNS}^{P_R=\mathbf{1}})_{ij} e^{i\gamma_j}|$ are independent of the phase parameters γ_j. The results are only dependent on the phase parameters α_i in P_L. Hereafter, for simplicity, we put $P_R = \mathbf{1}$.

Let us show that the neutrino oscillation parameters $\sin^2\theta_{13}$ and $\sin^2 2\theta_{23}$ are only dependent on a relative phase parameter $\alpha \equiv \alpha_3 - \alpha_2$. Since $(U_{PMNS})_{i3}$ is expressed as

$$(U_{PMNS})_{i3} = \sum_k (V_{CKM})_{ki}^* e^{-i\alpha_k} (U_{TB}^0)_{k3} = \frac{1}{\sqrt{2}} \left[-(V_{CKM})_{2i}^* e^{-i\alpha_2} + (V_{CKM})_{3i}^* e^{-i\alpha_3} \right], \tag{16}$$

the values $|(U_{PMNS})_{i3}|$ are dependent only on the parameter α. We illustrate the behaviors of $\sin^2\theta_{13}$ and $\sin^2 2\theta_{23}$ versus α in Fig.1 and Fig.2, respectively. Here, for numerical evaluation, we have used the Wolfenstein parameterization [12] of V_{CKM} and the best-fit values [13] $\lambda = 0.2272$, $A = 0.818$, $\overline{\rho} = 0.221$ and $\overline{\eta} = 0.340$. We find that the values $\sin^2\theta_{13}$ and $\sin^2 2\theta_{23}$ are almost insensitive to the value α, and those take $\sin^2\theta_{13} = 0.024 - 0.028$ and $\sin^2 2\theta_{23} = 0.94 - 0.95$. Those values are consistent with the present experimental data. As shown in Fig.1, if we take the result $\sin^2\theta_{13} = 0.016 \pm 0.010$ (1σ) obtained from a global analysis of neutrino oscillation data by Fogli *et al.* [5], we can obtain allowed bounds for α. The sizable value $\sin^2\theta_{13}$ is within a reach of forthcoming neutrino experiments planning by Double Chooz, Daya Bay, RENO, OPERA, and so on. The value $\sin^2 2\theta_{23} = 0.94 - 0.95$ is consistent with the present observed value [4] $\sin^2 2\theta_{23} = 1.00_{-0.13}$, and the predicted value will also be testable soon by precision measurements in solar and reactor neutrino experiments.

Previously, Plentinger and Rodejohann [10] have predicted possible deviations from tribimaximal mixing by assuming a specific form of the neutrino mass matrix and by assuming a CKM-like hierarchy of the mixing angles ($\theta^e_{12} = \lambda$, $\theta^e_{23} = A\lambda^2$, $\theta^e_{13} = B\lambda^3$) in the charged lepton sector. Furthermore, they have assumed the quark-lepton-complementarity (QLC) [11], and put an ad hoc relation $\theta^e_{12} = \theta_C$ (θ_C is the Cabibbo mixing angle). Then, they have obtained a relation

$$|(U_{PMNS})_{13}| \simeq \frac{1}{\sqrt{2}}|(V_{CKM})_{us}|. \tag{17}$$

Their result (17) agrees with our result $\sin^2\theta_{13} = 0.024 - 0.028$, because

$$|U_{(MNS)13}|^2 = \frac{1}{2}\left|(V_{CLM})^*_{cd} - (V_{CKM})^*_{td}e^{-i\alpha}\right|^2 \simeq \frac{1}{2}|(V_{CKM})_{us}|^2 \simeq 0.025, \tag{18}$$

from Eq.(16).

On the other hand, for the value $\tan^2\theta_{12}$, there is no simple situation (one-parameter dependency). The values $(U_{PMNS})_{11}$ and $(U_{PMNS})_{12}$ are given by

$$(U_{PMNS})_{11} = \frac{1}{\sqrt{6}}\left[2(V_{CKM})^*_{11}e^{-i\alpha_1} - (V_{CKM})^*_{21}e^{-i\alpha_2} - (V_{CKM})^*_{31}e^{-i\alpha_3}\right], \tag{19}$$

$$(U_{PMNS})_{12} = \frac{1}{\sqrt{3}}\left[(V_{CKM})^*_{11}e^{-i\alpha_1} + (V_{CKM})^*_{21}e^{-i\alpha_2} + (V_{CKM})^*_{31}e^{-i\alpha_3}\right], \tag{20}$$

so that the values $|(U_{PMNS})_{11}|$ and $|(U_{PMNS})_{12}|$ depend not only on $\beta \equiv \alpha_2 - \alpha_1$ but also on $\alpha \equiv \alpha_3 - \alpha_2$. However, since the observed CKM matrix parameters show $1 \gg |(V_{CKM})_{cd}|^2 \gg |(V_{CKM})_{td}|^2$, we can neglect the terms $(V_{CKM})^*_{31}e^{-i\alpha_3}$ compared with $(V_{CKM})^*_{11}e^{-i\alpha_1}$ and $(V_{CKM})^*_{21}e^{-i\alpha_2}$, so that the value $\tan^2\theta_{12}$ approximately depends on only the parameter β. We illustrate the behavior of $\tan^2\theta_{12}$ versus $\beta \equiv \alpha_2 - \alpha_1$ in Fig.3, in which we take typical values of α such as $\alpha = 0$ and $\alpha = -2\pi/3$. We can see that $\tan^2\theta_{12}$ is, in fact, insensitive to the parameter α. In contrast to the cases of $\sin^2\theta_{13}$ and $\sin^2 2\theta_{23}$, the value of $\tan^2\theta_{12}$ are highly sensitive to the parameter β as shown by

$$|(U_{PMNS})_{12}| \simeq \frac{1}{\sqrt{3}}\left[1 - |(V_{CKM})_{us}|\cos\beta\right], \tag{21}$$

from Eq.(20). The similar result has been obtained by Plentinger and Rodejohann [10]. The value of $\tan^2\theta_{12}$ takes from 0.24 to 1.00 according to the variation in β. In order to fit the observed value [3] $\tan^2\theta_{12} \simeq 0.5$, we must take $\beta \simeq \pm\pi/2$. This will put a constraint on scenarios which give a tribimaximal mixing.

Note that, from the relation (4), we can obtain a CP violating observable

$$J^\nu_{CP} \simeq -\frac{1}{6}|(V_{CKM})_{us}|\sin\beta, \tag{22}$$

as well as in a model given in Ref.[10]. Therefore, if we require a maximal CP violation in the lepton sector, we obtain $\beta \simeq \pm\pi/2$ as pointed out in Ref.[10], which is compatible with the constraint from the observed value $\tan^2\theta_{12} \simeq 0.5$.[14]

3 Summary

In conclusion, under the ansatz "$U_{PMNS} \to U_{TB}$ in the limit of $V_{CKM} \to \mathbf{1}$", we have speculated a simple relation $U_{PMNS} = V_{CKM}^\dagger U_{TB}$. We have not referred an explicit mechanism (model) which gives such a CKM mixing $V_{CKM} = \mathbf{1}$ in the limit of $U_{PMNS} = U_{TB}$. For example, a model [10] by Plentinger and Rodejohann is one of mass matrix models which explicitly realize our ansatz because they have put an ad hoc assumption $\sin\theta_{12}^e = \sin\theta_C$. A model [9] by Datta is also one of such models. However, such a model-building is not a purpose of the present paper. We have started our investigation by admitting the relation $U_{PMNS} \to U_{TB}$ as $V_{CKM} \to \mathbf{1}$ as an ansatz. The relation $U_{PMNS} = V_{CKM}^\dagger U_{TB}$ is widely valid for all models which are consistent with our ansatz.

By using the observed CKM matrix parameters, we have estimated the lepton mixing parameters $\sin^2\theta_{13}$, $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$. The values of $\sin^2 2\theta_{23}$ and $\sin^2\theta_{13}$ are almost independent of the phase convention, and they take values $\sin^2\theta_{13} = 0.024 - 0.028$ and $\sin^2 2\theta_{23} = 0.94 - 0.95$. The sizable value of $\sin^2\theta_{13}$ is within a reach of forthcoming neutrino experiments planning by Double Chooz, Daya Bay, RENO, OPERA, and so on. The value of $\sin^2 2\theta_{23}$ is also testable soon by precision measurements in solar and reactor neutrino experiments. On the other hand, the value of $\tan^2\theta_{12}$ has highly depended on the phase convention of the tribimaximal mixing, and the value has been in a range $0.24 < \tan^2\theta_{12} < 1.00$. Note that the phase matrix P_L cannot be absorbed into the rephasing of V_{CKM}, although it seems to be possible from the expression (4). Since the present observed value of $\tan^2\theta_{12}$ is $\tan^2\theta_{12} \simeq 0.5$, the phase parameter β is constrained as $\beta \simeq \pm\pi/2$. This put a strong constraint on models which lead to the exact tribimaximal mixing (2). The requirement of a maximal CP violation in the lepton sector is interestingly related to the observed value $\tan^2\theta_{12} \simeq 0.5$.

If the predicted values $\sin^2\theta_{13} = 0.024 - 0.028$ and $\sin^2 2\theta_{23} \simeq 0.94 - 0.95$ are denied by forthcoming neutrino oscillation experiments, it means a denial of the simple view that the lepton mixing U_{PMNS} becomes the exact tribimaximal mixing U_{TB} in the limit of $V_{CKM} \to \mathbf{1}$. We will be compelled to consider that the view stated above is oversimplified and the situation of quark and lepton flavor mixings is more complicated. The observed values of neutrino oscillation parameters will provide us a promising clue to a possible structure of U_{ed}, although we simply assumed $U_{ed} = \mathbf{1}$ in the expression (12). This will shortly become clear by forthcoming experiments.

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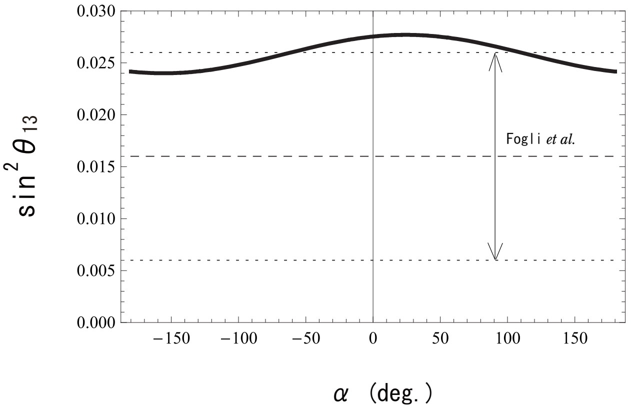

Fig. 1 Behavior of $\sin^2\theta_{13}$ versus $\alpha = \alpha_3 - \alpha_2$. The horizontal dashed and dotted lines denote the analysis $\sin^2\theta_{13} = 0.016 \pm 0.010$ (1σ) by Fogli *et al.* [5].

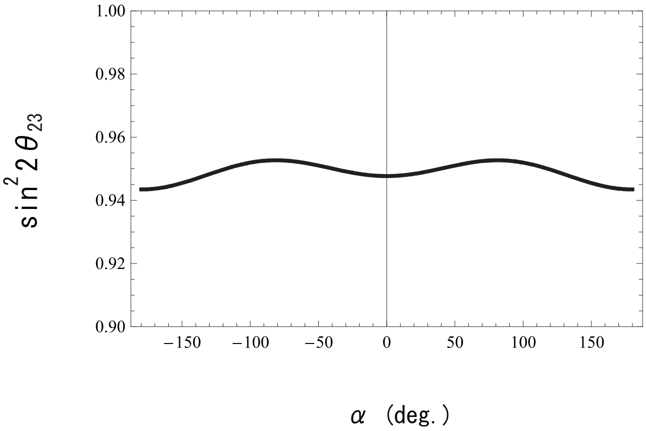

Fig. 2 Behavior of $\sin^2 2\theta_{23}$ versus $\alpha = \alpha_3 - \alpha_2$. The predicted value is consistent with the observed data [4] $\sin^2 2\theta_{23} = 1.00_{-0.13}$.

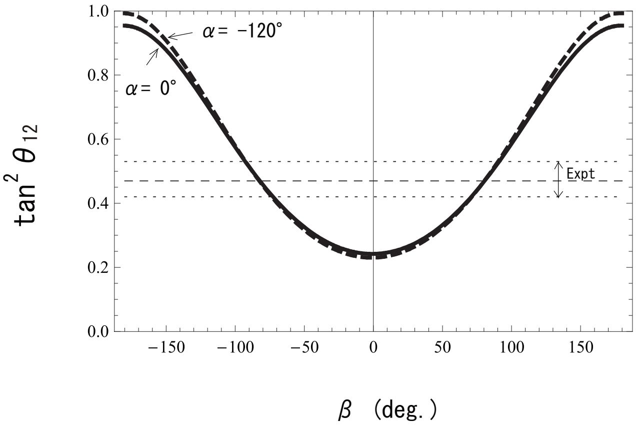

Fig. 3 Behavior of $\tan^2\theta_{12}$ versus $\beta = \alpha_2 - \alpha_1$. The horizontal dashed and dotted lines denote the observed values [3] $\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$.