Maximal CP Violation Hypothesis and a Lepton Mixing Matrix

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Abstract

Maximal CP violation hypothesis is applied to a simple lepton mixing matrix form $U = V^\dagger_{CKM} U_{TB}$, which has recently been speculated under an ansatz that U becomes an exact tribimaximal mixing U_{TB} in a limit of the quark mixing matrix $V_{CKM} \to \mathbf{1}$. The prediction $\tan^2\theta_{12} = 1/2$ in the case of the exact tribimaximal mixing $U = U_{TB}$ is considerably spoiled in the speculated mixing $U = V^\dagger_{CKM} U_{TB}$. However, the application of the hypothesis to the lepton sector can again recover the spoiled value to $\tan^2\theta_{12} \simeq 1/2$ if the original Kobayashi-Maskawa phase convention for V_{CKM} is adopted.

1 Introduction

Recently, an interesting form of the lepton mixing matrix U has been proposed [1]:

$$U = V^\dagger U_{TB}, \tag{1.1}$$

which was speculated under an ansatz that U becomes an exact tribimaximal mixing [2] U_{TB} in a limit $V \to \mathbf{1}$ (V is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix). Here, U_{TB} is given by

$$U_{TB} = P^\dagger(\delta_\ell) U^0_{TB} P(\gamma), \tag{1.2}$$

where

$$P(\delta) = \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}), \quad P(\gamma) = \mathrm{diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3}), \tag{1.3}$$

$$U^0_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1.4}$$

The ansatz which has derived the relation (1.1) is as follows: the up-quark and neutrino mass matrices in the limit of $U_u \to \mathbf{1}$ are given by $M^0_u = D_u$ and $M^0_\nu = U_{TB} D_\nu U^T_{TB}$ ($D_f = \mathrm{diag}(m_{f1}, m_{f2}, m_{f3})$), and those in the observed world with a realistic small deviation $V \neq \mathbf{1}$ from $V = \mathbf{1}$ become modified as $M^0_u \to M_u = U_u D_u U^\dagger_u$ and $M^0_\nu \to U_u M^0_\nu U^\dagger_u$ (we use a mass matrix convention $U^\dagger_f M_f U_f = D_f$). Therefore, we obtain $U_\nu = U_u U_{TB}$ and $U = U^\dagger_e U_\nu = U^\dagger_e U_d V^\dagger U_{TB}$. The ansatz "$U_d \to \mathbf{1}$ and $U_e \to \mathbf{1}$ in the limit of $U_u \to \mathbf{1}$" demands approximately $U_e = U_d$. (For an explicit neutrino mass matrix model which gives the relation (1.1), see, for example, [3, 4].)

The pure tribimaximal mixing $U = U_{TB}$ predicts $\tan^2\theta_{12} = 1/2$, $\sin^2 2\theta_{23} = 1$ and $\sin^2\theta_{13} = 0$ even if we consider a degree of freedom due to the phase convention given by (1.2), while those predictions in the lepton mixing matrix $U = V^\dagger U_{TB}$ are spoiled by the presence of $P(\delta_\ell)$. Especially, the strict prediction $\tan^2\theta_{12} = 1/2$ is considerably spoiled by the presence of a phase parameter $\beta \equiv \delta_{\ell 2} - \delta_{\ell 1}$: The predicted deviations of $\sin^2 2\theta_{23}$ and $\sin^2\theta_{13}$ from those in the exact tribimaximal mixing $U = U_{TB}$ are small, i.e. $0.024 \le \sin^2\theta_{13} \le 0.028$ and $0.094 \le \sin^2 2\theta_{23} \le 0.95$ depending on a phase parameter $\alpha \equiv \delta_{\ell 3} - \delta_{\ell 2}$, while the prediction $\tan^2\theta_{12} = 1/2$ becomes vague, i.e. $0.24 \le \tan^2\theta_{12} \le 1.00$ depending on the phase parameter β (see Fig.3 in Ref.[1]). (Note that the phase parameters γ_i, which are the so-called Majorana phases, do not affect neutrino oscillation phenomena.)

On the other hand, if we take $\beta \simeq \pi/2$, we can again predict $\tan^2\theta_{12} \simeq 1/2$. This was pointed out by Plentinger and Rodejohann [3], and also by the authors [1]. However, it is not clear whether the choice $\beta = \pi/2$ means really a case of the maximal CP violation or not, because there are three CP violating phases in the present scenario, i.e. α, β and δ_q (δ_q is a CP violating phase parameter in the CKM matrix $V(\delta_q)$). Usually, the maximal CP violation hypothesis is defined as follows: the nature takes values of CP violating phases so that a magnitude of the rephasing invariant quantity J [5] takes its maximal value. Therefore, the results under this definition of the maximal CP violation hypothesis depend on phase conventions of the flavor mixing matrix [6]. For example, in the standard expression [7] $V_{SD}(\delta_{SD})$ and original Kobayashi-Maskawa (KM) expression [8] $V_{KM}(\delta_{KM})$ of V, the rephasing invariant quantity J is given by

$$J_{SD} = c_{13}^2 s_{13} c_{12} s_{12} c_{23} s_{23} \sin\delta_{SD}, \tag{1.5}$$

and

$$J_{KM} = c_1 s_1^2 c_2 s_2 c_3 s_3 \sin\delta_{KM}, \tag{1.6}$$

respectively. Here, V_{SD} and V_{KM} are explicitly given by

$$V_{SD} = R_1(\theta_{23}) P_3(\delta_{SD}) R_2(\theta_{13}) P_3^\dagger(\delta_{SD}) R_3(\theta_{12})$$

$$= \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{SD}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{SD}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{SD}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{SD}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{SD}} & c_{23}c_{13} \end{pmatrix}, \tag{1.7}$$

$$V_{KM} = R_1^T(\theta_2) P_3(\delta_{KM} + \pi) R_3(\theta_1) R_1(\theta_3)$$

$$= \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta_{KM}} & c_1c_2s_3 + s_2c_3e^{i\delta_{KM}} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta_{KM}} & c_1s_2s_3 - c_2c_3e^{i\delta_{KM}} \end{pmatrix}, \tag{1.8}$$

respectively, where

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_3(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.9}$$

$$P_3(\delta) = \mathrm{diag}(1,\ 1,\ e^{i\delta}), \tag{1.10}$$

$s = \sin\theta$ and $c = \cos\theta$. It is well known [6] that the standard expression $V_{SD}(\delta_{SD})$ with $\delta_{SD} = \pm\pi/2$ cannot describe the observed CKM matrix parameters, while $V_{KM}(\delta_{KM})$ with $\delta_{KM} = \pm\pi/2$ can well describe the observed those. (In the standard phase convention $V_{SD}(\delta_{SD})$, a case with $\delta_{SD} \simeq 70^\circ$ is in favor of the observed data.)

In this paper, we assume the maximal CP violation hypothesis not only for the quark sector, but also for the lepton sector. In the present scenario, since the matrix $V(\delta_q)$ in Eq.(1.1) is already fixed by the observed data in the quark sector, the rephasing invariant quantity J is only a function of α and β. In Sec.2, we re-investigate the CKM mixing parameters from the data in the quark sector. In Sec.3, we will apply the maximal CP violation hypothesis to the lepton sector. Since, in Sec.2, we can do a reasonably good fitting of the data only in the case $V = V_{KM}$ under the maximal CP violation hypothesis, we adopt $V = V_{KM}$ in the mixing $U = V^\dagger U_{TB}$ in the lepton sector, too. We find that the maximal value of $|J(\alpha,\beta)|$ takes place at $\beta \simeq \pm\pi/2$ and $\alpha \simeq 0$ (or $\alpha \simeq \pi$), so that we can again obtain $\tan^2\theta_{12} \simeq 1/2$. Finally, Sec.4 is devoted to the summary and concluding remarks.

2 Maximal CP violation hypothesis in the quark sector

First, we estimate the CKM matrix parameters in the original KM matrix $V_{KM}(\delta_{KM})$ without assuming the maximal CP violation. Using input values [9] $|V_{us}| = 0.2255 \pm 0.0019$, $|V_{ub}| = 0.00393 \pm 0.00036$ and $|V_{td}| = 0.0081 \pm 0.0006$, we obtain the rotation parameters

$$s_1 = 0.2255 \pm 0.0019, \quad s_2 = 0.0359^{+0.0030}_{-0.0029}, \quad s_3 = 0.0174^{+0.0018}_{-0.0017}. \tag{2.1}$$

We fit the value of δ_{KM} to the observed value $|V_{cb}| = 0.0412 \pm 0.0011$, and thereby, we obtain $\delta_{KM} = (84^{+16}_{-22})^\circ$.

Inversely, if we assume the maximal CP violation, i.e. $\delta_{KM} = \pm\pi/2$, we can fix the parameters s_1, s_2 and s_3 from the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$, and can predict the value of $|V_{td}|$. (Although the value of s_2 is readily fixed from the relation $V_{td} = s_1 s_2$ in the original KM matrix, we use the value $|V_{cb}|$ as the input value, because the accuracy of $|V_{td}|$ is not so precise compared with that of $|V_{cb}|$.) For convenience, we define $V_{us} > 0$, so that we take $s_1 = -\sqrt{|V_{us}|^2 + |V_{ub}|^2} < 0$ and $s_3 = V_{ub}/\sqrt{|V_{us}|^2 + |V_{ub}|^2}$. We also define that all c_i $(i = 1, 2, 3)$ are positive, i.e. $c_i = \sqrt{1 - s_i^2} > 0$. For input values [9], $V_{us} = 0.2255 \pm 0.0019$, $V_{ub} = -s_1 s_3 = \pm(0.00393 \pm 0.00036)$, $|V_{cb}| = 0.0412 \pm 0.0011$, we obtain reasonable CKM parameter values only for cases with $s_3 s_\delta / s_2 > 0$ $(s_\delta = \sin\delta_{KM} = \pm 1)$:

$$|s_2| = 0.0376^{+0.0019}_{-0.0021}, \quad |V_{td}| = 0.0085 \pm 0.0005, \tag{2.2}$$

$$\phi_1 = (24.4^{-3.5}_{+3.2})^\circ, \quad \phi_2 = (89.963 \mp 0.004)^\circ, \quad \phi_3 = (65.7^{+3.1}_{-3.5})^\circ, \tag{2.3}$$

where the angles ϕ_i of the unitary triangle have been defined by

$$\phi_1 = \arg\left(-\frac{V_{21}V_{23}^*}{V_{31}V_{33}^*}\right), \quad \phi_2 = \arg\left(-\frac{V_{31}V_{33}^*}{V_{11}V_{13}^*}\right), \quad \phi_3 = \arg\left(-\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*}\right). \tag{2.4}$$

Those predicted values are in agreement with the observed CKM matrix data [9].

If we adopt $V = V_{SD}(\delta_{SD})$, by using the global fit values, $|V_{us}| = 0.2257 \pm 0.0010$, $|V_{cb}| = 0.0415^{+0.0010}_{-0.0011}$, $|V_{ub}| = 0.00359 \pm 0.00016$ and $|V_{td}| = 0.00874^{+0.00026}_{-0.00037}$, which have been reported by Particle Data Group [9], we obtain

$$\delta_{SD} = (68.9^{+\ 9.1}_{-10.7})^\circ . \tag{2.5}$$

Thus, for the standard phase convention V_{SD}, we cannot demand the maximal CP violation hypothesis consistently, because the value $\delta_{SD} = (68.9^{+\ 9.1}_{-10.7})^\circ$ is far from the value $\delta_{SD} = \pi/2$ in the maximal CP violation hypothesis.

3 Maximal CP violation hypothesis in the lepton sector

Next, we calculate the rephasing invariant quantity J in the lepton sector from the relation

$$J = \mathrm{Im}(U_{23} U_{12} U_{22}^* U_{13}^*), \tag{3.1}$$

where

$$\begin{aligned}
U_{12} &= \tfrac{1}{\sqrt{3}} \left(V_{11}^* e^{-i\delta_{\ell 1}} + V_{22}^* e^{-i\delta_{\ell 2}} + V_{32}^* e^{-i\delta_{\ell 3}} \right) e^{i\gamma_2}, \\
U_{22}^* &= \tfrac{1}{\sqrt{3}} \left(V_{12} e^{i\delta_{\ell 1}} + V_{21} e^{i\delta_{\ell 2}} + V_{31} e^{i\delta_{\ell 3}} \right) e^{-i\gamma_2}, \\
U_{23} &= \tfrac{1}{\sqrt{2}} \left(-V_{22}^* e^{-i\delta_{\ell 2}} + V_{32}^* e^{-i\delta_{\ell 3}} \right) e^{i\gamma_3}, \\
U_{13}^* &= \tfrac{1}{\sqrt{2}} \left(-V_{21} e^{i\delta_{\ell 2}} + V_{31} e^{i\delta_{\ell 3}} \right) e^{-i\gamma_3}.
\end{aligned} \tag{3.2}$$

We obtain

$$J \simeq \frac{1}{6} s_1 (s_\beta + s_2 s_\alpha c_\beta - s_2 c_\alpha s_\beta + 2 s_3 c_\alpha c_\beta s_\delta), \tag{3.3}$$

where $\alpha = \delta_{\ell 3} - \delta_{\ell 2}$, $\beta = \delta_{\ell 2} - \delta_{\ell 1}$, $s_\delta = \sin\delta_{KM} = \pm 1$, and $c_\alpha = \cos\alpha$ and so on, and we have used the observed fact $1 \gg |s_1| \simeq |V_{us}| \gg |s_2| \simeq |V_{td}|/|V_{us}| \sim |s_3| \simeq |V_{ub}|/|V_{us}|$. The value J is approximately given by $J \simeq (1/6)\sin\theta_1 \sin\beta$, so that the maximal CP violation hypothesis demands $\beta \simeq \pm\pi/2$. More precisely speaking, from $\partial J/\partial\beta = 0$, we obtain

$$\cot\beta \simeq 2 s_3 c_\alpha s_\delta + s_2 s_\alpha . \tag{3.4}$$

Similarly, we obtain

$$\tan\alpha \simeq \frac{s_2 c_\beta}{2 s_3 c_\beta s_\delta - s_2 s_\beta}, \tag{3.5}$$

from $\partial J/\partial\alpha = 0$ (but with a rough approximation). Since $\beta \simeq \pm\pi/2$ from Eq.(3.4), we obtain $\alpha \simeq 0$ or π from Eq.(3.5). We should note that the maximal CP violation hypothesis can determine values of the phase parameters α and β simultaneously. The numerical results obtained with use of no approximation are given in Table 1. As an example of the behavior of $|J(\alpha,\beta)|$, J versus α in a typical case $(s_\delta, s_3, s_2) = (+,-,-)$ in Table 1 is illustrated in Fig. 1. As seen in Table 1, the value of α takes 0 or π according as $s_2 < 0$ or $s_2 > 0$, i.e. $V_{td} < 0$ or $V_{td} > 0$. For comparison, we show the results for the case of $V = V_{SD}$ in Table 2. In this case, we obtain $\alpha \simeq 25^\circ$, although we can still obtain $\beta \simeq \pi/2$.

s_δ	s_3	s_2	$(\pm J)_{max}$	α	β
$+$	$+$	$+$	0.03772 ± 0.00037	$-(175.6^{-1.1}_{+0.4})^\circ$	$-(87.93 \mp 0.21)^\circ$
			$-(0.03800^{-0.00037}_{+0.00034})$	$+(179.3^{+0.6}_{-1.3})^\circ$	$+(91.88^{+0.20}_{-0.22})^\circ$
$+$	$-$	$-$	0.03772 ± 0.00037	$+(3.64^{+0.46}_{-0.44})^\circ$	$-(87.96 \mp 0.21)^\circ$
			$-(0.03801^{-0.00035}_{+0.00034})$	$+(2.86^{+0.29}_{-0.30})^\circ$	$+(92.01^{+0.21}_{-0.20})^\circ$
$-$	$-$	$+$	0.03772 ± 0.00037	$-(175.6^{-1.1}_{+0.4})^\circ$	$-(87.93 \mp 0.21)^\circ$
			$-(0.03800^{-0.00037}_{+0.00034})$	$+(179.3^{+0.6}_{-1.3})^\circ$	$+(91.88^{+0.20}_{-0.22})^\circ$
$-$	$+$	$-$	0.03772 ± 0.00037	$+(3.64^{+0.46}_{-0.44})^\circ$	$-(87.96 \mp 0.21)^\circ$
			$-(0.03801^{-0.00035}_{+0.00034})$	$+(2.86^{+0.29}_{-0.30})^\circ$	$+(92.01^{+0.21}_{-0.20})^\circ$

Table 1: Predicted values of CP violating phase factors $\alpha = \delta_{\ell 3} - \delta_{\ell 2}$ and $\beta = \delta_{\ell 2} - \delta_{\ell 1}$ under the maximal CP violation hypothesis.

$(\pm J)_{max}$	α	β
$+(0.0378 \pm 0.0002)$	$+(25.4^{+2.4}_{-3.6})^\circ$	$-(88.2^{+0.1}_{-0.2})^\circ$
$-(0.0381 \pm 0.0002)$	$+(24.5^{+1.8}_{-1.7})^\circ$	$+(91.8 \pm 0.1)^\circ$

Table 2: Predicted values of CP violating phase factors $\alpha = \delta_{\ell 3} - \delta_{\ell 2}$ and $\beta = \delta_{\ell 2} - \delta_{\ell 1}$ for the case $V = V_{SD}(\delta_{SD})$ with $\delta_{SD} = (68.9^{+\ 9.1}_{-10.7})^\circ$.

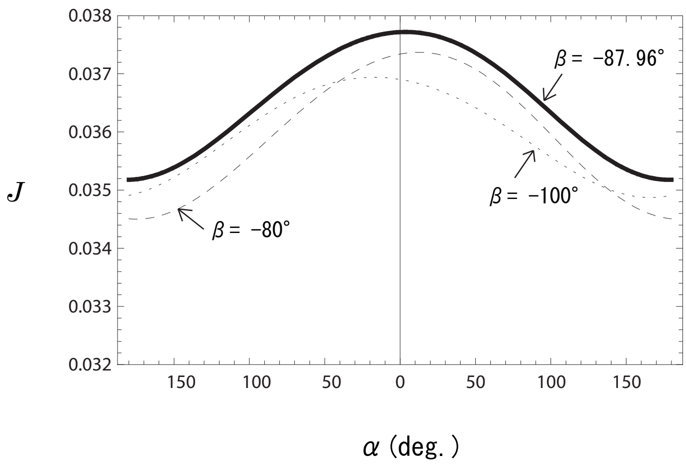

Fig. 1 An example of the behavior of $J(\alpha, \beta)$ versus α for typical values of β. The case corresponds to the case with $(s_\delta, s_3, s_2) = (+, -, -)$ in Table 1.

4 Summary

In conclusion, we have applied a maximal CP violation hypothesis to a simple lepton mixing matrix form $U = V^\dagger U_{TB}$, which has recently been speculated under an ansatz that U becomes an exact tribimaximal mixing U_{TB} in a limit of the quark mixing matrix $V \to \mathbf{1}$. The mixing matrix U_{TB} includes two phase parameters $\alpha = \delta_{\ell 3} - \delta_{\ell 2}$ and $\beta = \delta_{\ell 2} - \delta_{\ell 1}$ due to the phase convention of the tribimaximal mixing. Therefore, the rephasing invariant quantity J in the lepton sector is a function of phase parameters α, β and δ_q (δ_q is a CP violating phase parameter in the quark mixing matrix $V(\delta_q)$). Since we have demanded the maximal CP

s_2	$\|J\|_{max}$	α	β	$\sin^2 2\theta_{23}$	$\sin^2\theta_{13}$	$\tan^2\theta_{12}$
$+$	$J>0$	$-(175.6^{-1.1}_{+0.4})^\circ$	$-(87.93\mp 0.21)^\circ$	0.944 ± 0.001	0.0273 ± 0.0005	0.507 ± 0.001
	$J<0$	$(179.3^{+0.6}_{-1.3})^\circ$	$(91.88^{+0.20}_{-0.22})^\circ$	0.944 ± 0.001	0.0273 ± 0.0005	0.530 ± 0.001
$-$	$J>0$	$+(3.64^{+0.46}_{-0.44})^\circ$	$-(87.96\mp 0.21)^\circ$	0.944 ± 0.001	0.0235 ± 0.0004	0.508 ± 0.002
	$J<0$	$+(2.86^{+0.29}_{-0.30})^\circ$	$+(92.01^{+0.21}_{-0.20})^\circ$	0.944 ± 0.001	0.0235 ± 0.0004	0.535 ± 0.002

Table 3: Predicted values of neutrino oscillation parameters under the maximal CP violation hypothesis in the case $V=V_{KM}(\delta_{KM})$ with $\delta_{KM}=\pm\pi/2$.

violation hypothesis for the quark sector, too, we have taken the original KM phase convention $V_{KM}(\delta_{KM})$ with $\delta_{KM}=\pm\pi/2$ as the CKM matrix V, because the standard phase convention $V=V_{SD}(\delta_{SD})$ with $\delta_{SD}=\pm\pi/2$ cannot reproduce the observed CKM parameters consistently. Then, the quantity J in the lepton sector is a function of only α and β. The requirement of the maximal $|J(\alpha,\beta)|$ fixes both the parameters α and β simultaneously. Note that the phase parameters α and β do not appear in J for the case $U=U_{TB}$, so that they are unobservable quantities in the limit of $V\to\mathbf{1}$, while those can become observable quantities in the case of the realistic mixing $U=V^\dagger U_{TB}$ with $V\neq\mathbf{1}$.

We have found that only for the case $V=V_{KM}$, the maximal CP violation hypothesis leads to interesting results, $\delta_{KM}=\pm\pi/2$ in the quark sector, and $\beta\simeq\pm\pi/2$ and $\alpha\simeq 0$ (or $\alpha\simeq\pi$) in the lepton sector. The result $\beta\simeq\pm\pi/2$ predicts [1] $\tan^2\theta_{12}\simeq 1/2$ which is in good agreement with the observed value $\tan^2\theta_{12}=0.47^{+0.06}_{-0.05}$ [10]. The result $\alpha\simeq 0$ (or $\alpha\simeq\pi$) means that the neutrino mass matrix $M_\nu^0=U_{TB}D_\nu U_{TB}^T$ in the limit of $V\to\mathbf{1}$ is nearly $2\leftrightarrow 3$ symmetric (antisymmetric). The predicted neutrino oscillation parameters are listed in Table 3.

It is worthwhile noticing that the neutrino mixing matrix $U=V^\dagger U_{TB}$ with the realistic $V\neq\mathbf{1}$ spoils the prediction $\tan^2\theta_{12}=1/2$ in the pure tribimaximal mixing $U=U_{TB}$ as $0.24\leq\tan^2\theta_{12}\leq 1.00$, while the maximal CP violation hypothesis fixes the phase parameter β as $\beta\simeq\pm\pi/2$, so that the hypothesis recovers the spoiled value of $\tan^2\theta_{12}$ to $\tan^2\theta_{12}\simeq 1/2$. The parameter β is fixed almost independently of the phase convention of the quark mixing matrix V, while the parameter α is fixed dependently on the phase convention of V: If we take $V=V_{KM}(\delta_{KM})$ with $\delta_{KM}=\pm\pi/2$ under the maximal CP violation hypothesis, we obtain the result $\alpha\simeq 0$ or π, while if we take $V=V_{SD}(\delta_{SD})$ with $\delta_{SD}=68.9^\circ$ (without the maximal CP violation hypothesis in the quark sector), we obtain $\alpha\simeq 25^\circ$, which does not seem to be a suggestive value. Thus, the maximal CP violation hypothesis can lead to phenomenologically interesting results not only in the quark sector, but also in the lepton sector. However, the reason why the hypothesis is so effective only when we take $V=V_{KM}$ has still not been understood. Also, theoretical ground for the maximal CP violation hypothesis has still been unclear. We hope that, by investigating these problems, one will find a promising clue to a unified mass matrix model.

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