O(3) Flavor Symmetry and an Empirical Neutrino Mass Matrix

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Abstract

Based on a new approach to quark and lepton masses, where the mass spectra originate in vacuum expectation values of O(3)-flavor **1+5** (gauge singlet) scalars, a neutrino mass matrix of a new type is speculated. The mass matrix is described in terms of the up-quark and charged lepton masses, and, by assuming a special flavor basis, it can be accommodated to a nearly tribimaximal mixing without explicitly assuming a discrete symmetry. Quark mass relations are also discussed based on the new approach.

1 Introduction

One of the most challenging problems in contemporary particle physics is to clarify the origin of flavors. For this purpose, searching for a unified description of the observed quark and lepton mass spectra will provide a promising clue to us. In conventional mass matrix model, the quark and lepton mass matrices M_f are given by the forms $(M_f)_{ij} = (Y_f)_{ij} v_H$, where $(Y_f)_{ij}$ are coupling constants of the Yukawa interactions $\bar{f}_{Li} f_{Rj} H^0$ and v_H is a vacuum expectation value (VEV) of the neutral component of the Higgs scalar H, $v_H = \langle H^0 \rangle$. Against this conventional approach, there is another idea: the origin of the mass spectra is due to VEV structures of Higgs scalars H_{ij} [1, 2], i.e. $(M_f)_{ij} = y_f \langle (H^0)_{ij} \rangle$. In the present paper, we will investigate an extended model by separating the role of H_{ij} into two roles: one of the roles is to cause $\mathrm{SU}(2)_L$ symmetry breaking at the energy scale $\mu \sim 10^2$ GeV, and the conventional $\mathrm{SU}(2)_L$ doublet Higgs scalars H_u and H_d still play the role in this scenario; another one is to give an origin of the mass spectra, and we consider gauge-singlet scalars $(Y_f)_{ij}$ whose VEVs give effective Yukawa coupling constants $\langle (Y_f)_{ij} \rangle / \Lambda$ (Λ is an energy scale of the effective theory). As a typical model with such gauge-singlet scalars $(Y_f)_{ij}$, there has been a model [3] with U(3)-nonet scalars, where quarks and leptons are assigned to **3** and $\bar{\mathbf{3}}$ of $\mathrm{U}(3)_F$, so that Y_f $(f = u, d, \nu, e)$ are assigned to nonet of $\mathrm{U}(3)_F$. However, this U(3) scenario with this assignment cannot apply to a grand unification theory (GUT) scenario, because, for example, in SU(5)-GUT, the $\mathrm{SU}(2)_L$ doublet and singlet quark fields Q and U should be assigned to the same multiplet **3**, so that Y_u must be $\bar{\mathbf{6}}$ (not nonet). Then, the model considerably become complicated, because we need fields **6** in addition to fields $\bar{\mathbf{6}}$ in order to make singlets of U(3).

In this paper, considering applicability of the scenario to a GUT scenario, we assume an O(3) flavor symmetry instead of U(3). We consider the following superpotential terms:

$$W_Y = \sum_{i,j} \frac{y_u}{\Lambda} U_i (Y_u)_{ij} Q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} D_i (Y_d)_{ij} Q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} L_i (Y_\nu)_{ij} N_j H_u + \sum_{i,j} \frac{y_e}{\Lambda} L_i (Y_e)_{ij} E_j H_d + h.c. + \sum_{i,j} y_R N_i (Y_R)_{ij} N_j, \tag{1.1}$$

where Y_f ($f = u, d, \nu, e$) and Y_R are O(3)-flavor **1**+**5** (gauge singlet) fields , and Q and L are quark and lepton SU(2)$_L$ doublet fields of O(3)$_F$ triplets, and U, D, N, and E are SU(2)$_L$ singlet matter fields of O(3)$_F$ triplets. Therefore, the fields Y_f and Y_R are symmetric. Our basic assumption is as follows: the fields Y_f and Y_R always behave as a combination of **1**+**5**, so that, for example, **5** alone never appears in the interaction terms. Hereafter, for convenience, we will denote **1**+**5** as **6**. (Note that if Y_f are composed of a single (**1**+**5**), Y_f are real, but if Y_f are composed of (**1**+**5**)'s more than two, Y_f can be complex with $Y_f^T = Y_f$.) In order to distinguish the fields Y_f from each other, we assign additional U(1) charges $Q_X(Y_f) = q_f$ to Y_f ($f = u, d, \nu, e$), and $Q_X(U) = -q_u$ to U, $Q_X(E) = -q_e$ to E, and so on. The field Y_R has the charge $Q_X(Y_R) = 2q_\nu$. In this paper, we will write down our superpotential W under the O(3)$_F$ and U(1)$_X$ symmetries.

In the present approach, we will investigate relations among Y_f and Y_R by using supersymmetric (SUSY) vacuum conditions for the superpotential $W = W_u + W_d + W_\nu + W_e + W_R + W_Y$, where W_f ($f = u, d, \nu, e$) and W_R determine the VEV structures of Y_f and Y_R, respectively. (Since we can easily show $\langle Q\rangle = \langle L\rangle = \langle U\rangle = \langle D\rangle = \langle N\rangle = \langle E\rangle = 0$, hereafter, we will drop the term W_Y from W when we investigate the VEV structures of Y_f.) Such an approach to quark and lepton mass matrices has first been adopted by Ma [4] and has been developed by the author within a context of U(3)-flavor nonet model [3]. In the conventional mass matrix approach, the investigation has now been on a level with theoretically reliable ground via a long period of phenomenological investigations. However, the present approach is still in its beginning stage, so that we need more phenomenological investigations. Therefore, we adopt the following strategy in this approach: (i) First, we search for a possible form of the superpotential W which can successfully provide relations among the observed masses and mixings from the phenomenological point of view; (ii) Next, we investigate what symmetries or quantum number assignments can explain such a specific form of W. In this paper, we will investigate a possible form of W by putting weight on the step (i).

Recently, as a byproduct in such approach, an interesting neutrino mass matrix form [5] has been reported: the form is given by

$$M_\nu \propto Y_e^{-1} Y_u^{1/2} + Y_u^{1/2} Y_e^{-1} + \xi_0 \mathbf{1}. \tag{1.2}$$

Neutrino mass matrix models which leads to the so-called tribimaximal neutrino mixing [6] have usually been proposed based on discrete symmetries, while, if we assume a specific relation between a diagonal basis of the charged lepton mass matrix (we call it "e-basis") and a diagonal basis of the down-quark mass matrix (we call it "d-basis"), the mass matrix (1.2) can be accommodate to a nearly tribimaximal neutrino mixing without explicitly assuming a discrete symmetry. On the other hand, in general, if a neutrino mass matrix M_ν can give reasonable masses and mixing, a neutrino mass matrix $\tilde{M}_\nu$ with an inverse form of M_ν, $\tilde{M}_\nu = m_0^2 M_\nu^{-1}$, can also give reasonable predictions, because, by taking the inverse of $U^\dagger M_\nu U^* = M_\nu^D \equiv \mathrm{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$, we can obtain $U^T \tilde{M}_\nu U = m_0^2 (M_\nu^D)^{-1} = \mathrm{diag}(m_0^2/m_{\nu 1}, m_0^2/m_{\nu 2}, m_0^2/m_{\nu 3})$, i.e. we obtain the mixing matrix U^* instead of U and neutrino masses $(m_0^2/m_{\nu 1}, m_0^2/m_{\nu 2}, m_0^2/m_{\nu 3})$ with a normal (inverse) hierarchy instead of neutrino masses $(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$ with an inverse (normal) hierarchy. Therefore, in this paper, instead of the model $M_\nu \propto Y_e^{-1} Y_u^{1/2} + Y_u^{1/2} Y_e^{-1} + \xi_0 \mathbf{1} =$

$Y_e^{-1}(Y_u^{1/2}Y_e + Y_eY_u^{1/2} + \xi_0 Y_eY_e)Y_e^{-1}$, we will investigate a neutrino mass matrix with a seesaw-type

$$M_\nu = \frac{y_\nu^2 v_{Hu}^2}{y_R\Lambda^2} Y_\nu Y_R^{-1} Y_\nu^T, \tag{1.3}$$

where Y_R and Y_ν are given by

$$Y_R \propto Y_u^{1/2}Y_e + Y_eY_u^{1/2} + \xi_0 Y_eY_e, \tag{1.4}$$

and $Y_\nu \propto Y_e$, respectively. In the model (1.2), the matrix M_ν was for Dirac neutrinos, while the present M_ν is for Majorana neutrinos. The mass matrix (1.2) could not provide a reasonable mass spectrum without adjusting the parameter ξ_0, while, in this paper, we will give a small value of $\Delta m^2_{21}/|\Delta m^2_{32}|$ without the ξ_0-term. Note that, in the present scenario, since the Dirac neutrino mass matrix Y_ν is identical with the charged lepton mass matrix Y_e, the nearly tribimaximal mixing originates in the structure of Y_R.

In the next section, we will derive the neutrino mass matrix (1.3) with the form (1.4) of Y_R by using SUSY vacuum conditions for an $\mathrm{O}(3)_F$ and $\mathrm{U}(1)_X$ invariant superpotential, and we will evaluate the mass matrix M_ν by using the observed values of up-quark and charged lepton masses. However, in order to evaluate the neutrino mixing matrix, we must know the form of (1.3) on the e-basis, especially the form of $Y_u^{1/2}$ on the e-basis. Therefore, in the present paper, we will put a phenomenological assumption on the relation between e- and d-bases. Then, we will find that, by using the observed Cabibbo-Kobayashi-Maskawa (CKM) matrix parameters, the mass matrix (1.3) with the form (1.4) can be accommodated to the observed nearly tribimaximal mixing. However, since the result is dependent on a phenomenological assumption on the form of $\langle Y_u \rangle$ on the basis "e-basis", the mass matrix is still an empirical one. Nevertheless, we consider that the result is very suggestive.

In Sec.3, we will discuss the structure of Y_d lightly. Finally, Sec.4 will be devoted to concluding remarks.

2 Neutrino mass matrix without a discrete symmetry

In order to give the operator $Y_u^{1/2}$ in the expression (1.4), we introduce additional $\mathrm{O}(3)_F$ **6** fields Φ_u and X_u with the $\mathrm{U}(1)_X$ charges $\frac{1}{2}q_u$ and $-q_u$, respectively. Then, we can write down the superpotential for the u-sector

$$W_u = \lambda_u \mathrm{Tr}[\Phi_u\Phi_u X_u] + m_u \mathrm{Tr}[Y_u X_u] + W_{\Phi u}(\Phi_u). \tag{2.1}$$

From SUSY vacuum conditions (for the moment, we regard W_u as W), we obtain

$$\frac{\partial W}{\partial X_u} = 0 = \lambda_u \Phi_u\Phi_u + m_u Y_u, \tag{2.2}$$

$$\frac{\partial W}{\partial Y_u} = 0 = m_u X_u, \tag{2.3}$$

$$\frac{\partial W}{\partial \Phi_u} = 0 = \lambda_u(\Phi_u X_u + X_u\Phi_u) + \frac{\partial W_{\Phi u}}{\partial \Phi_u}. \tag{2.4}$$

From the condition (2.2), we obtain a bilinear relation

$$\langle Y_u \rangle = -\frac{\lambda_u}{m_u} \langle \Phi_u \rangle \langle \Phi_u \rangle, \tag{2.5}$$

so that the field Φ_u plays a role of $Y_u^{1/2}$. However, since the matrix $\langle \Phi_u \rangle$ is not Hermitian, the relation

$$U_u^T \langle Y_u \rangle U_u = \langle Y_u \rangle^D \propto \text{diag}(m_{u1}, m_{u2}, m_{u3}), \tag{2.6}$$

does not always mean

$$U_u^T \langle \Phi_u \rangle U_u = \langle \Phi_u \rangle^D \propto \text{diag}(\sqrt{m_{u1}}, \sqrt{m_{u2}}, \sqrt{m_{u3}}), \tag{2.7}$$

where D denotes that the matrix is on its diagonal basis. As we see later, we need the relation (2.7). Therefore, we assume the field Φ_u (and also Y_f) is real, so that the matrix U_u is orthogonal matrix.

From the condition (2.3), we obtain

$$\langle X_u \rangle = 0. \tag{2.8}$$

Therefore, from the condition (2.4), we obtain $\partial W_{\Phi u}/\partial \Phi_u = 0$. We assume that three eigenvalues of $\langle \Phi_u \rangle$ can completely be determined by this condition $\partial W_{\Phi u}/\partial \Phi_u = 0$. However, for this purpose, the superpotential term $W_{\Phi u}$ will include $\text{U}(1)_X$ symmetry breaking terms. In this paper, we do not discuss the explicit form of $W_{\Phi u}$. We assume that the VEV values are suitably given by Eq.(2.7) with the observed up-quark masses m_{ui}.

For convenience, for the e-sector, we also assume superpotential term W_e similar to the u-sector:

$$W_e = \lambda_e \text{Tr}[\Phi_e \Phi_e X_e] + m_e \text{Tr}[Y_e X_e] + W_{\Phi e}(\Phi_e), \tag{2.9}$$

where Φ_e, X_e and Y_e have $\text{U}(1)_X$ charges $\frac{1}{2}q_e$, $-q_e$ and q_e, respectively, so that we obtain relations

$$Y_e = -\frac{\lambda_e}{m_e} \Phi_e \Phi_e, \tag{2.10}$$

with $\Phi_e^D \propto \text{diag}(\sqrt{m_{e1}}, \sqrt{m_{e2}}, \sqrt{m_{e3}})$, where we have again assumed that the field Φ_e is real. (Hereafter, for simplicity, we will sometimes express VEV matrices $\langle A \rangle$ as simply A.)

In order to obtain the relation $Y_\nu \propto Y_e$, we assume the following structure of W_ν:

$$W_\nu = \lambda_\nu \phi_\nu \text{Tr}[Y_\nu X_\nu] + \lambda_{\nu e} \phi_e \text{Tr}[Y_e X_\nu], \tag{2.11}$$

where ϕ_ν and ϕ_e are gauge- and flavor-singlet fields, and we assign $\text{U}(1)_X$ charges as $Q_X(X_\nu) = x_\nu$, $Q_X(\phi_\nu) = -(q_\nu + x_\nu)$ and $Q_X(\phi_e) = -(q_e + x_\nu)$. From $\partial W/\partial \phi_\nu = 0$ and $\partial W/\partial \phi_e = 0$, we obtain $X_\nu = 0$. From $\partial W/\partial X_\nu = 0$, we obtain

$$Y_\nu = -\frac{\lambda_{\nu e} \phi_e}{\lambda_\nu \phi_\nu} Y_e. \tag{2.12}$$

Next, let us investigate a possible form of W_R. In order to obtain the relation (1.4) from the phenomenological point of view, we assume the following form of W_R:

$$W_R = \lambda_R \mathrm{Tr}[(Y_e \Phi_u + \Phi_u Y_e) X_R] + m_R \mathrm{Tr}[Y_R X_R], \tag{2.13}$$

where we have assumed U(1)$_X$ charges $Q_X(Y_R) = -Q_X(X_R) = 2q_\nu$ and $\frac{1}{2}q_u + q_e - 2q_\nu = 0$, From SUSY vacuum conditions $\partial W/\partial Y_R = 0$, we obtain $X_R = 0$. Then, the requirement $\partial W/\partial Y_e = 0$ leads to the condition $\partial W_e/\partial Y_e = 0$, so that we obtain the relation (2.10). From $\partial W/\partial X_R = 0$, we obtain

$$Y_R = -\frac{\lambda_R}{m_R}(Y_e \Phi_u + \Phi_u Y_e). \tag{2.14}$$

Thus, we can obtain the desirable form (1.4) of Y_R (without the ξ_0-term).

For convenience, let us define a name of a flavor basis as follows: when a VEV matrix $\langle Y_f \rangle$ takes a diagonal form on a basis, we call the basis "f-basis", and we denote a form of a matrix $\langle A \rangle$ on the f-basis as $\langle A \rangle_f$. In order to obtain the neutrino mixing matrix form on the e-basis, we must know a matrix form of $\langle \Phi_u \rangle$ on the e-basis, i.e. $\langle \Phi_u \rangle_e$, although the form $\langle \Phi_u \rangle_u$ on the u-basis is given by Eq.(2.7). Let us defined a transformation of a VEV matrix $\langle Y_f \rangle$ from a b-basis to an a-basis as

$$\langle Y_f \rangle_a = U_{ba}^T \langle Y_f \rangle_b U_{ba}, \tag{2.15}$$

where U_{ab} are unitary matrices, and they satisfy $U_{ab}^\dagger = U_{ba}$ and $U_{ab} U_{bc} = U_{ac}$. (These operators U_{ab} are not always members of O(3) flavor transformations.) Since $Y_f^T = Y_f$ in the present model, the VEV matrix $\langle Y_f \rangle$ are diagonalized as $U_f^T \langle Y_f \rangle U_f = \langle Y_f \rangle^D$. Therefore, $\langle Y_u \rangle_d$ is given by $\langle Y_u \rangle_d = V^T(\delta) \langle Y_u \rangle_u V(\delta)$, where $V(\delta)$ is the standard expression of CKM matrix. The simplest assumption is to consider that the d-basis is identical with the e-basis, so that we can regard U_{ue} as $U_{ue} = V$ because $U_{ud} = V$. Then, we can evaluate the neutrino mass matrix (1.3) with $\langle Y_R \rangle_e \propto \langle \Phi_u \rangle_e \langle Y_e \rangle_e + \langle Y_e \rangle_e \langle \Phi_u \rangle_e$ by using the form

$$\langle \Phi_u \rangle_e = U_{ue}^T \langle \Phi_u \rangle_u U_{ue} = V^T(\delta) \langle \Phi_u \rangle^D V(\delta). \tag{2.16}$$

(Note that the O(3)-invariant relation (2.5) is not valid on the d-basis, because $U_{ud} = V(\delta)$ is not orthogonal, although we can still use $\langle \Phi_u \rangle_d = V^T(\delta) \langle \Phi_u \rangle_u V(\delta)$. The relation (2.5) is valid only on a basis which is transformed from the u-basis by an orthogonal transformation.) In the numerical calculation of M_ν, we adopt the standard phase convention $V(\delta)$ [8] of the CKM matrix, and use the following input values: the up-quark masses [7] at the energy scale $\mu = M_Z$, $m_{u1} = 0.00127$ GeV, $m_{u2} = 0.619$ GeV, $m_{u3} = 171.7$ GeV, and the CKM parameters [8], $|V_{us}| = 0.2257$, $|V_{cb}| = 0.0416$, $|V_{ub}| = 0.00431$. (Here, we have used the quark mass values at $\mu = M_Z$ because we have used the CKM parameter values at $\mu = M_Z$. For the energy scale dependency of the mass ratios and CKM parameters, for example, see Ref.[9].) As seen in Table 1, the results are dependent on the CP violating phase parameter δ. The present experimental data [8] on the CKM matrix favor $\delta \simeq \pi/3$. However, as seen in Table 1, the predicted value of $\sin^2 2\theta_{23}$ at $\delta \simeq \pi/3$ is in poor agreement with the observed value $\sin^2 2\theta_{23} = 1.00_{-0.13}$ [10], although the predicted value of $\tan^2 \theta_{12}$ is roughly in agreement with the observed value

Table 1: The δ dependency of predicted values in the case $U_{ue} = V(\delta)$. The values of $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$ are estimated by $\sin^2 2\theta_{23} = 4|(U_\nu)_{23}|^2|(U_\nu)_{33}|^2$ and $\tan^2\theta_{12} = |(U_\nu)_{12}/(U_\nu)_{11}|^2$, respectively. The numerical results in the case $U_{ue} = V(-\delta)$ are identical with the case $U_{ue} = V(\delta)$.

δ	$\sin^2 2\theta_{23}$	$\tan^2\theta_{12}$	$\lvert U_{13}\rvert$	$\Delta m^2_{21}/\Delta m^2_{32}$
0	0.3890	0.4679	0.01156	0.00220
60°	0.7702	0.4979	0.01779	0.00100
90°	0.9237	0.5228	0.01529	0.00070
120°	0.9836	0.5434	0.01055	0.00063
180°	0.9998	0.5604	0.00034	0.00062

$\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$ [11]. Therefore, we cannot regard that the e-basis is identical with the d-basis.

However, as seen in Table 1, note that the case with $\delta \geq 2\pi/3$ can give a nearly tribimaximal mixing. Especially, we notice that the case $\delta = \pi$ highly realizes the tribimaximal mixing. Since we have assumed that Φ_u and Φ_e (therefore, Y_u and Y_e) are real matrices, so that the u- and e-bases are connected each other by an orthogonal transformation. This guarantees to use the relations (2.5), (2.12) and (2.14), which are obtained from the O(3)-invariant superpotential, on the u- and e-bases. If we still suppose $U_{ue} \simeq U_{ud} = V(\delta)$, the possible candidates of the orthogonal matrix U_{ue} will be $U_{ue} = V(0)$ and/or $U_{ue} = V(\pi)$. (Indeed, we can show [5] $U_{ed} = U_{ue}^\dagger U_{ud} = V^\dagger(\delta_{ue})V(\delta) = \mathbf{1} + \mathcal{O}(|V_{ub}|)$, so that we can still consider $U_{ue} \simeq U_{ud}$). Therefore, the case $U_{ue} = V(\pi)$ is likely. However, in this paper, we a priori assume the form $U_{ue} = V(\delta_{ue})$ with $\delta_{ue} \geq 2\pi/3$ as a phenomenological requirement suggested in Table 1. Again, we summarize our phenomenological neutrino mass matrix which can lead to a nearly tribimaximal mixing for $|\delta_{ue}| \geq 2\pi/3$ as follows:

$$(M_\nu)_e = k_\nu Y_e^D \left[\left(V^T(\delta_{ue})\Phi_u^D V(\delta_{ue})\right) Y_e^D + Y_e^D \left(V^T(\delta_{ue})\Phi_u^D V(\delta_{ue})\right)\right]^{-1} Y_e^D, \tag{2.17}$$

where $Y_e^D \propto \mathrm{diag}(m_e, m_\mu, m_\tau)$ and $\Phi_u^D \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$. (For the phenomenological reason why the mass matrix (2.17) can give a nearly tribimaximal mixing, see Ref.[5].)

As seen in Table 1, the predicted value of $R = \Delta m^2_{21}/\Delta m^2_{32}$ is considerably small compared to the observed value $|R| = 0.028 \pm 0.004$, where we have used the observed values $\Delta m^2_{21} = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$ [11] and $|\Delta m^2_{32}| = (2.74^{+0.44}_{-0.26}) \times 10^{-3}\,\mathrm{eV}^2$ [10]. The value R can be adjusted by taking the ξ_0-term in Eq.(1.4) into consideration. (It is easy to bring the ξ_0-term into the present model.) However, the smallness of Δm^2_{21} can also become mild by considering the renormalization group equation (RGE) effects. Since, so far, we have not fixed the energy scale Λ, the values without RGE effects have been listed in Table 1. The RGE effects will be able to give a reasonable value of R without the ξ_0 term. By the way, the present neutrino masses are normal hierarchical, so that, if regard $m_{\nu 3}$ as $m_{\nu 3} = \sqrt{\Delta m^2_{32}} = 0.0523$ eV, we can obtain neutrino masses $m_{\nu 1} = 0.78$ meV, $m_{\nu 2} = 1.52$ meV and $m_{\nu 3} = 52.3$ meV for the case $\delta_{ue} = \pi$.

3 Down-quark sector

So far, we did not discuss a structure of W_d. Although the purpose of the present paper is not to give a structure of W_d, here, let us discuss a possible structure of W_d lightly. As we have assumed that the fields Φ_u and Φ_e are real and since we know that the CP is broken in the quark sector, we must consider that Y_d is complex. By way of trial, let us assume the following superpotential W_d:

$$W_d = \lambda_{du} \left(\mathrm{Tr}[\Phi_X]\mathrm{Tr}[\Phi_u X_d] + e^{i\alpha}\mathrm{Tr}[\Phi_u]\mathrm{Tr}[\Phi_X X_d] \right) + m_d \mathrm{Tr}[Y_d X_d] + \lambda_d \, \mathrm{det}\Phi_X, \tag{3.1}$$

where we have assumed U(1)$_X$ charges $Q_X(X_d) = -Q_X(Y_d) \equiv -q_d$ and $Q_X(\Phi_X) = q_d - \frac{1}{2}q_u$. Since we consider Φ_u and Φ_X are real, the factor $e^{i\alpha}$ in Eq.(3.1) has been introduced by hand in order to yield a CP violating phase. Under this charge assignment, the term $\mathrm{Tr}[\Phi_X \Phi_u X_d]$ is also allowed. So far, in W_u, W_e and W_R, we have not considered cubic terms of a type $\mathrm{Tr}[A]\mathrm{Tr}[BC]$, while, in W_d, we have assumed such a cubic term $\mathrm{Tr}[A]\mathrm{Tr}[BC]$ instead of a cubic term $\mathrm{Tr}[ABC]$. At present, the form (3.1) is merely a phenomenological assumption, and the form (3.1) is not a general form. Also note that the cubic term $\mathrm{det}\Phi_X$ breaks the U(1)$_X$ symmetry. From the condition $\partial W/\partial \Phi_X = 0$, we obtain

$$Y_d = -\frac{\lambda_{du}}{m_d}(\mathrm{Tr}[\Phi_X]\Phi_u + e^{i\alpha}\mathrm{Tr}[\Phi_u]\Phi_X). \tag{3.2}$$

Since we have already taken $\partial W_u/\partial \Phi_u = 0$ in Eq.(2.4), we obtain $X_d = 0$ for $\Phi_X \neq 0$ from the condition $\partial W/\partial \Phi_u = \lambda_{du}(\mathrm{Tr}[\Phi_X]X_d + e^{i\alpha}\mathrm{Tr}[\Phi_X X_d]\mathbf{1}) + \partial W_u/\partial \Phi_u = 0$. Then, from the condition $\partial W/\partial \Phi_X = 0$, we obtain

$$0 = \frac{\partial \mathrm{det}\Phi_X}{\partial \Phi_X} = \Phi_X \Phi_X - \mathrm{Tr}[\Phi_X]\Phi_X + (1/2)\left(\mathrm{Tr}^2[\Phi_X] - \mathrm{Tr}[\Phi_X \Phi_X]\right)\mathbf{1}, \tag{3.3}$$

where we have used a formula for a 3×3 Hermitian matrix A, $\mathrm{det}A = (1/3)\mathrm{Tr}[AAA] - (1/2)\mathrm{Tr}[AA]\mathrm{Tr}[A] + (1/6)\mathrm{Tr}^3[A]$. The constraint (3.3) demands that the matrix $\langle \Phi_X \rangle$ is a rank-1 matrix. Such a rank-1 matrix is generally expressed as $(\langle \Phi_X \rangle_u)_{ij} = v_X x_i x_j$, where x_i are real and $x_1^2 + x_2^2 + x_3^2 = 1$. Therefore, $\langle Y_d \rangle_u$ is expressed as

$$(\langle Y_d \rangle_u)_{ij} \propto \delta_{ij}\sqrt{m_{ui}} + e^{i\alpha} x_i x_j (\sqrt{m_{u3}} + \sqrt{m_{u2}} + \sqrt{m_{u1}}), \tag{3.4}$$

so that we obtain

$$\frac{m_s}{m_b} \simeq \frac{1}{2}\sqrt{\frac{m_c}{m_t}}, \tag{3.5}$$

where we have assumed $e^{i\alpha} \simeq 1$ and $(x_2/x_3)^2 \ll \sqrt{u_{u2}/m_{u3}}$. The observed values are $m_s/m_b \simeq 0.019$ and $\sqrt{m_c/m_t} \simeq 0.060$ at $\mu = M_Z$ [7], so that the relation (3.5) is in roughly agreement with the observed value. (We can adjust the predicted value to the observed value by taking a suitable choice of x_i and α.) Also we can obtain $m_d/m_s \simeq \sqrt{m_u/m_c}$, but the result is sensitive to the values of x_i/x_j and α, so that we do not discuss no more details of m_{di}/m_{dj} in this paper.

4 Concluding remarks

In conclusion, we have proposed a new approach to the masses and mixings of quarks and leptons. In the new approach, we write a superpotential W for O(3)-flavor $\mathbf{1}+\mathbf{5}$ fields Y_f whose VEVs give effective Yukawa coupling constants $\langle (Y_f)_{ij}\rangle/\Lambda$ and we obtain relations among masses and mixings from the SUSY vacuum conditions. In this approach, we cannot predict the absolute values of the masses and mixings, but we can obtain relations among the VEV matrices Y_f and Y_R. Under this approach, we have found an empirical neutrino mass matrix (2.17). The form (2.17) was found as a byproduct when we assumed that $\langle Y_e\rangle$ and $\langle Y_d\rangle$ can simultaneously be diagonalized. Regrettably, the idea $U_{ue} = U_{ud} = V(\delta)$ with $\delta \simeq \pi/3$ was failed to explain the observed fact $\sin^2 2\theta_{23} \simeq 1$, but we have found that $U_{ue} = V(\delta_{ue})$ with $\delta_{ue} \geq 2\pi/3$ can successfully give the observed values $\sin^2 2\theta_{23} \simeq 1$ and $\tan^2\theta_{12} \simeq 0.5$. At present, there is no theoretical reason for the form $U_{ue} = V(\delta_{ue})$. (Since we have assumed that Φ_u and Φ_e are real, U_{ue} must be real.) Nevertheless, it is worthwhile noticing because the form is of a new type which is related to the up-quark masses and which successfully leads to the nearly tribimaximal mixing without assuming any discrete symmetry. (However, we do not consider that this denies applicability of a discrete symmetry to the neutrino sector. Rather, we consider that this suggests that the discrete symmetry is applicable not only to the lepton sector, but also to the quark sector.)

Since the present approach is still in its beginning stage, we have many tasks to investigate: for example, (i) investigation of the explicit structures of $W_{\Phi u}(\Phi_u)$ and $W_{\Phi e}(\Phi_e)$, which completely determine the eigenvalues of $\langle\Phi_u\rangle$ and $\langle\Phi_e\rangle$, i.e. $(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$ and $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, respectively; (ii) investigation of the explicit structure of W_d in order to give more definite quark mass relations and CKM matrix parameters (Y_d in this paper has included free parameters x_i, so that we cannot derive definite conclusions because we can adjust the parameters x_i to the observed values freely); (iii) investigation of symmetries and quantum number assignments which can uniquely derive the present specific (phenomenological) form of W. In the present scenario, most of the fields Φ_u, Φ_e, Y_f $(f = u, d, e)$, Y_R, and so on, take VEV of the order of Λ, and their masses are also of the order Λ. However, some components of those fields are massless in the SUSY limit, and, under the SUSY breaking at $\mu \sim 1$ TeV, they have masses of the order $\mu \sim 1$ TeV. Since those particles are gauge singlets, in principle, they are harmless in the low energy phenomenology. However, in TeV region physics, we may expect fruitful phenomenology about flavor-mediated (but gauge-singlet) processes. This approach will shed a new light on the quark and lepton masses and mixings and on a TeV scale flavor physics.

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